H28 年国公２次試験専門問題解説（水理）

【No.20】

(1)特性曲線法を用いた非定常流解析の問題.

(a)

$$\frac{\partial h}{\partial t}+v\frac{\partial h}{\partial x}+\frac{A}{B}\frac{\partial v}{\partial x}=0 \qquad ③$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+g\frac{\partial h}{\partial x}=g\left(i-I_f\right) \qquad ②$$

今,

$$c=\sqrt{g\frac{A}{B}}$$

とおくと,

$$\frac{\partial c}{\partial t}=\sqrt{g}\frac{\partial\sqrt{A/B}}{\partial t}=\frac{1}{2}\sqrt{\frac{gB}{A}}\frac{\partial}{\partial t}\left(\frac{A}{B}\right)=\frac{1}{2}\sqrt{\frac{gB}{A}}\left(\frac{\partial h}{\partial t}-\frac{A}{B^2}\frac{\partial B}{\partial t}\right) \quad \therefore\frac{\partial h}{\partial t}=2\sqrt{\frac{A}{gB}}\frac{\partial c}{\partial t}+\frac{A}{B^2}\frac{\partial B}{\partial t}=\frac{2c}{g}\frac{\partial c}{\partial t}+\frac{A}{B^2}\frac{\partial B}{\partial t}$$

$$\frac{\partial c}{\partial x}=\frac{1}{2}\sqrt{\frac{gB}{A}}\left(\frac{\partial h}{\partial x}-\frac{A}{B^2}\frac{\partial B}{\partial x}\right) \quad \therefore\frac{\partial h}{\partial x}=\frac{2c}{g}\frac{\partial c}{\partial x}+\frac{A}{B^2}\frac{\partial B}{\partial x}$$

これらを，③，②式へ代入すると,

$$\frac{2c}{g}\frac{\partial c}{\partial t}+\frac{A}{B^2}\frac{\partial B}{\partial t}+v\left(\frac{2c}{g}\frac{\partial c}{\partial x}+\frac{A}{B^2}\frac{\partial B}{\partial x}\right)+\frac{A}{B}\frac{\partial v}{\partial x}=0$$

$$\therefore\ \frac{\partial c}{\partial t}+\frac{c}{2B}\frac{\partial B}{\partial t}+v\frac{\partial c}{\partial x}+v\frac{c}{2B}\frac{\partial B}{\partial x}+\frac{c}{2}\frac{\partial v}{\partial x}=0 \qquad (B)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+g\left(\frac{2c}{g}\frac{\partial c}{\partial x}+\frac{A}{B^2}\frac{\partial B}{\partial x}\right)=g\left(i-I_f\right)$$

$$\therefore\ \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+2c\frac{\partial c}{\partial x}+\frac{c^2}{B}\frac{\partial B}{\partial x}=g\left(i-I_f\right) \qquad (A)$$

A+2B：

$$\frac{\partial(v+2c)}{\partial t}+v\frac{\partial(v+2c)}{\partial x}+c\frac{\partial(v+2c)}{\partial x}+\frac{c}{B}\left(\frac{\partial B}{\partial t}+(v+c)\frac{\partial B}{\partial x}\right)=g\left(i-I_f\right)$$

ここで,

$$\frac{\partial B}{\partial t}+(v+c)\frac{\partial B}{\partial x}$$

は，速度 $v+c$ で移動する座標系で見ると不変となり，0 とおける.（→要検証）

よって,

$$\frac{\partial(v+2c)}{\partial t}+(v+c)\frac{\partial(v+2c)}{\partial x}=g\left(i-I_f\right)$$

同様に，A-2B は,

$$\frac{\partial(v-2c)}{\partial t}+(v-c)\frac{\partial(v-2c)}{\partial x}=g\left(i-I_f\right)$$

よって，題意と比較すると，

$$\frac{A}{B}\frac{dK}{dh}=c, K=2c$$

が満足される必要がある．ゆえに，

$$\frac{A}{B}\frac{dK}{dh}=\frac{2A}{B}\frac{dc}{dh}=\frac{2A}{B}\sqrt{g}\frac{d}{dh}\sqrt{\frac{A}{B}}=c\frac{d}{dh}\left(\frac{A}{B}\right)=c\left(1-\frac{A}{B^2}\frac{dB}{dh}\right)$$

となり，$\mathrm{d}B/\mathrm{d}h$ は小さいと仮定する必要がある．（→要検証）

よって，

ア：

$$\sqrt{g\frac{B}{A}}$$

イ：

$$\sqrt{g\frac{A}{B}}$$

となる．

(b)

（ア）となる．理由は，不連続帯の伝搬速度が$\sqrt{gh}-v$であることから，水深 h に依存して変化し，上流側の水深が深い方が浅い部分より速度が速くなるためである．

(2)

(a)

①に $A=Bh$ を代入すると，

$$\frac{\partial(Bh)}{\partial t}+\frac{\partial(vBh)}{\partial x}=0 \quad \therefore\frac{\partial h}{\partial t}+\frac{\partial(vh)}{\partial x}=0$$

同様に②に h を掛け，⑦に v を掛けたものを加えると，

$$h\frac{\partial v}{\partial t}+v\frac{\partial h}{\partial t}+vh\frac{\partial v}{\partial x}+v\frac{\partial(hv)}{\partial x}+gh\frac{\partial h}{\partial x}=gh\left(i-I_f\right)$$

$$\therefore\ \frac{\partial(vh)}{\partial t}+\frac{\partial}{\partial x}\left(hv^2+\frac{1}{2}gh^2\right)=gh\left(i-I_f\right)$$

(b)⑦式について，

$$\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}\frac{\partial h}{\partial t}dx=\lim_{\varepsilon\to+0}\left\{\frac{\partial}{\partial t}\int_{x_b-\varepsilon}^{x_b+\varepsilon}h\,dx-\frac{dx_b}{dt}[h]_{x_b+\varepsilon}+\frac{dx_b}{dt}[h]_{x_b-\varepsilon}\right\}=-\frac{dx_b}{dt}h_2+\frac{dx_b}{dt}h_1$$

$$\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}\frac{\partial(vh)}{\partial x}dx=\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}d(vh)=[vh]_{x_b+\varepsilon}-[vh]_{x_b-\varepsilon}=v_2h_2-v_1h_1$$

$$\therefore\ -\frac{dx_b}{dt}h_2+\frac{dx_b}{dt}h_1+v_2h_2-v_1h_1=0$$

同様に⑧式について，

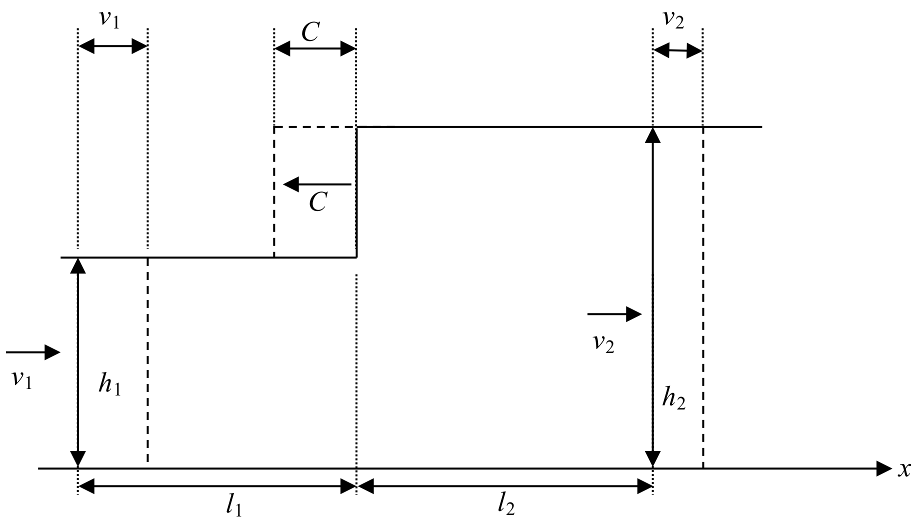
$$\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}\frac{\partial(vh)}{\partial t}dx=\lim_{\varepsilon\to+0}\left\{\frac{\partial}{\partial t}\int_{x_b-\varepsilon}^{x_b+\varepsilon}vh\,dx-\frac{dx_b}{dt}[vh]_{x_b+\varepsilon}+\frac{dx_b}{dt}[vh]_{x_b-\varepsilon}\right\}=-\frac{dx_b}{dt}v_2h_2+\frac{dx_b}{dt}v_1h_1$$

$$\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}\frac{\partial}{\partial x}\left(hv^2+\frac{1}{2}gh^2\right)dx=\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}d\left(hv^2+\frac{1}{2}gh^2\right)=h_2v_2^2+\frac{1}{2}gh_2^2-h_1v_1^2-\frac{1}{2}gh_1^2$$

$$\lim_{\varepsilon\to+0}\int_{x_b-\varepsilon}^{x_b+\varepsilon}gh\left(i-I_f\right)dx=\lim_{\varepsilon\to+0}\left\{gi\int_{x_b-\varepsilon}^{x_b+\varepsilon}hdx-g\int_{x_b-\varepsilon}^{x_b+\varepsilon}hI_f\,dx\right\}=0$$

$$\therefore\ -\frac{dx_b}{dt}v_2h_2+\frac{dx_b}{dt}v_1h_1+h_2v_2^2+\frac{1}{2}gh_2^2-h_1v_1^2-\frac{1}{2}gh_1^2=0$$

(c)



図に示すような，定常流の段波を考え，単位時間後に破線の位置に移動する場合に，質量保存則を用いると，

$$l_1Bh_1+l_2Bh_2=Bh_1(l_1-v_1-C)+Bh_2(l_2+v_2+C)$$

$$\therefore\ h_1(v_1+C)=h_2(v_2+C)$$

ここで，$C=-\mathrm{d}x_b/\mathrm{d}t$ とおけるので，(b)で得られた第一式と一致することが分かる．

同様に，運動量保存則から，

$$\rho\{Bh_1(l_1-v_1-C)v_1+Bh_2(l_2+v_2+C)v_2-Bh_1\,l_1v_1-Bh_2l_2v_2\}=B\int_0^{h_1}p_1dy-B\int_0^{h_2}p_2dy$$

圧力は静水圧をとるので，底面から鉛直上向きに y 軸をとると，

$$p_i=\rho g(h_i-y)$$

となる．

$$B\rho g\int_{h_i}(h_i - y)dh_i = \frac{B\rho g h_i^2}{2}$$

$$\therefore h_1(-v_1 - C)v_1 + h_2(v_2 + C)v_2 = \frac{gh_1^2}{2} - \frac{gh_2^2}{2}$$

よって，(b)で得られた第二式と一致することが分かる．

(d)

$$h_1(v_1 + C) = h_2(v_2 + C)$$

より，

$$v_2 = \frac{h_1}{h_2}(C + v_1) - C = C\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2}$$

これを，二式目に代入すると，

$$h_1(-v_1 - C)v_1 + h_2\left(C\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2} + C\right)\left(C\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2}\right) = \frac{gh_1^2}{2} - \frac{gh_2^2}{2}$$

整理すると，

$$C^2\left\{h_1\left(\frac{h_1}{h_2} - 1\right)\right\} + 2h_1\left(\frac{h_1}{h_2} - 1\right)v_1 C + h_1\left(\frac{h_1}{h_2} - 1\right)v_1^2 - \frac{gh_1^2}{2} + \frac{gh_2^2}{2} = 0$$

$$C = -v_1 \pm \frac{1}{2A}\sqrt{2Agh_2^2\left(\frac{h_1^2}{h_2^2} - 1\right)}$$

ここで，

$$A = h_1\left(\frac{h_1}{h_2} - 1\right)$$

また，

$$C = -\frac{dx_b}{dt}$$

より，

$$\frac{dx_b}{dt} = v_1 \mp \frac{1}{2A}\sqrt{2Agh_2^2\left(\frac{h_1^2}{h_2^2} - 1\right)} = v_1 \mp \frac{\sqrt{gh_2^2\left(\frac{h_1^2}{h_2^2} - 1\right)}}{\sqrt{2h_1\left(\frac{h_1}{h_2} - 1\right)}} = v_1 \mp \sqrt{\frac{gh_2^2}{2h_1}\left(\frac{h_1}{h_2} + 1\right)} = v_1 \mp \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}$$

また，

$$v_2 = C\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2}$$

より，

$$v_2 = -\frac{dx_b}{dt}\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2} = \left\{-v_1 \pm \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}\right\}\left(\frac{h_1}{h_2} - 1\right) + v_1\frac{h_1}{h_2} = v_1 \pm \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}\left(\frac{h_1}{h_2} - 1\right)$$

(e)

不連続帯は上流向きに伝播するので，

$$\frac{dx_b}{dt} = v_1 - \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}$$

よって，v_2は，

$$v_2 = v_1 + \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}\left(\frac{h_1}{h_2} - 1\right) = v_1 - \sqrt{\frac{gh_2}{2}\left(1 + \frac{h_2}{h_1}\right)}\left(1 - \frac{h_1}{h_2}\right)$$