SKIN FRICTION OF TAPER TYPED PILE RELATED TO
SOIL COMPRESSIBILITY

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ABSTRACT: In this study, a mobilized mechanism of skin friction in the taper shaped piles is first discussed in connection with the degree of the taper and soil compressibility. And then, a simple and practically useful estimating method for predicting the skin friction of a taper type pile is proposed based on the geometrical and theoretical considerations in terms of cavity expansion theory. Further, the effectiveness of the model is verified by comparing with the results from the model pile load tests.

1. INTRODUCTION

When a taper shape pile is set up and is then penetrated in a sandy ground, the ground surrounding the pile is moved and pressed outward to the horizontal direction against the pile, where, such phenomenon is called as "pressing effect" in this study. It is considered that the pressing effect gives a significant influence on the mobilization of the skin friction, in which both soil compressibility and shear stiffness may become key factors. Authors have already reported that the pressing effect on the skin friction is remarkable through the model pile load tests (Yasufuku et al., 2004). It is desired to present a rational method for predicting the skin friction of the taper shaped pile by properly introducing the pressing effect of the pile. In this study, the pressing effect of a taper shaped pile during vertical penetration into a sandy ground is first discussed related to the degree of the taper angle and soil compressibility surrounding the pile. A practically useful and rational evaluation method of skin friction of a taper shaped pile in sandy soil ground is presented based on an extended cavity expansion theory, together with. Further, the effectiveness of the method is verified by comparing with the results from the model pile load tests.

2. BASIC IDEA FOR EVALUATING THE VERTICAL BEARING CAPACITY

Specification for Highway Bridge gives a following equation as an estimating method of the ultimate pile bearing capacity based on the results of the field and laboratory investigations (JRA, 1996):

\[ R_c = U \sum L_i f_i + q_i A \]  \hspace{1cm} (1)

where \( R_c \): ultimate bearing capacity of pile, \( A \): pile tip area, \( q_i \): pile end bearing capacity, \( U \): pile circumference, \( L_i \): thickness in each layer, \( f_i \): maximum skin friction of pile. The first and second terms are related to the skin friction of pile and pile-tip bearing capacity, respectively. However, the main part of the vertical bearing capacity of a pile is often mobilized from the skin friction in practical designs within the limits of allowable displacement, because relatively large displacements are needed to mobilize the end bearing capacity. Thus, it becomes more important to rationally and precisely evaluate the skin friction from a geomechanical point of view.

3. EVALUATION OF SKIN FRICTION FOR TAPER SHAPED PILE

3.1 Basic equations

Skin friction of a pile is generally determined as the sum of pile to soil cohesion and friction components as shown in the following equation:

\[ f_s = c_s' + p_a \tan \phi_s' \]  \hspace{1cm} (2)

where \( f_s \) is the skin friction at unit length of a pile, \( c_s' \) and \( \phi_s' \) are the adhesion and friction parameters between pile and soil, and \( p_a \) is the effective horizontal stress acting on the pile. It was considered that the mobilized mechanism of skin friction between pile and soil was essentially due to the shear failure in a thin layer of soils surrounding the pile, which corresponds to a sufficiently large strain level. When assuming such a mechanism, it was reasonable and rational to use the strength parameters at the critical state condition such that (Yasufuku et al., 1997):

\[ c_s' = 0 \hspace{1cm} (3a) \]

\[ \phi_s' = \phi_v \hspace{1cm} (3b) \]

where, \( \phi_v \) is a friction angle at the critical state, which is defined as the condition such that a large shear strain is produced without any changes of mean effective principal stresses, shear stresses and volumetric strains. If this friction angle is used for the soil, it must be uniquely determined irrespective of density and overburden pressure, which may give some advantages for practical use. When applying this idea to eq.(2), the skin friction of the taper shaped pile
is given by

\[ f_s = \frac{p_w}{\tan \phi_{vr}} \]  \hspace{1cm} (4)

\( p_w \) is then represented by an effective vertical stress and the coefficient of horizontal earth pressure \( K \) such that:

\[ p_w = K \sigma_{v}^{'} + \Delta \sigma_{v0}^{'} \]  \hspace{1cm} (5)

where \( K_0 \) is the coefficient of earth pressure at rest and \( \Delta \sigma_{v0}^{'} \) is an expected increment of horizontal stresses when a taper pile is penetrated. Thus, in order to estimate the skin friction in eq.(5), it is needed to precisely evaluate the stress increment of the taper shaped pile. In this study, the stress increment is discussed as a boundary issue of an expanded cylindrical cavity schematically shown in Fig.1. The cavity expansion theory presented by Vesic(1972) is extended to evaluate the horizontal stress \( p_w \) which includes the stress increment due to the penetration of the taper shaped pile. Introducing eq.(5) into eq.(4), the skin friction, together with the coefficient of horizontal stress related to the stress increment and the \( p_w \) are given by

\[ f_s = \left( 1 + \frac{\Delta \sigma_{v0}^{'}}{\sigma_{v0}^{'}} \right) K_0 \sigma_{v}^{'}, \tan \phi_{vr} = K \sigma_{v}^{'}, \tan \phi_{vr} \]  \hspace{1cm} (6)

where,

\[ K = \left( 1 + \frac{\Delta \sigma_{v0}^{'}}{\sigma_{v0}^{'}} \right) K_0 \]  \hspace{1cm} (7)

3.2 Incremental horizontal stress \( p_w \) related to soil compressibility

Soil ground is first assumed as a homogeneous body with an isotropic elastic-perfectly plastic material. As shown in Fig.1, let's consider a cylindrical cavity, which is cylindrical extended from the inner diameter \( R_c \) of the cavity to \( R_e \). Then, the \( p_w \) in Fig.1 can be estimated by solving the boundary value problem using the Vesic's theory. In his theory, \( R_w \) is generally assumed as zero. In this study, the magnitude of \( R_w \) is however assumed to be a finite value, which is a key assumption for obtaining a realistic value of \( p_w \) which can explain the penetration effect of the taper shaped pile. This assumption is believed to be an extension of his theory. The derivation process of \( p_w \) is almost same as that of the original Vesic theory (1972) except for assuming that \( R_w \) is a finite value. The result of the derivation is summarized as follows:

\[ p_w = K \sigma_{v}^{'} \]  \hspace{1cm} (8a)

\[ K = \left[ 1 + \sin \phi_{vr} \right] \left[ 1 + \frac{G}{K_0 \sigma_{v}^{'} \tan \phi_{vr}} \right] \]  \hspace{1cm} (8b)

\[ I_w = \frac{\mu - \Delta_{w} \sec \phi_{vr}}{1 + \Delta_{w} \sec \phi_{vr}} \]  \hspace{1cm} (8c)

\[ \mu = \frac{\sin \phi_{vr}}{1 + \sin \phi_{vr}} \]  \hspace{1cm} (8d)

\[ K_0 = 1 - \sin \phi_{vr} \]  \hspace{1cm} (8e)

\[ \beta = 1 + \Delta_{w} - \alpha^2 \]  \hspace{1cm} (8f)

where, \( \mu \) and \( I_w \) are reduced rigidity index and rigidity index of soil, respectively; \( G \) shear stiffness of soil; \( \Delta_{w} \) average volumetric strain for the plastic zone around a cavity; \( K_0 \) coefficient of earth pressure at rest related to Jakay's equation; \( \alpha \) penetration factor. Based on the numerical analysis using the idea from Baligh(1976), \( \alpha \), reflecting the average compressibility of the ground surrounding the pile is strongly related to the \( I_w \) in eq.(8c), which has been already reported by Yasufuku et al.(2001). The following empirical equation is proposed:

\[ \Delta_{w} = \frac{5}{0.1 + \mu} \]  \hspace{1cm} (9)

In addition, the penetration factor is directly defined as

\[ \alpha = \frac{R_t}{R_w} \]  \hspace{1cm} (10)

which directly presents the degree of the expansion effect due to the pile penetration. When \( \beta = 1.0 \) in eq.(8f), eq.(8) reduces to the Vesic's formula. Four experimental parameters \( \phi_{vr}, \mu, \sigma_{v}^{'} \), and \( \alpha \) are included to evaluate the skin friction of taper shaped pile.

3.3 Characteristics of the coefficient of horizontal pressure \( K \) normalized by \( K_0 \)

The typical relationship between \( K/K_0 \) given in eq.(8) and \( \alpha \) is shown in Figs.2(a) and (b). The results are compared with each other, paying attention to \( \phi_{vr}^{'} \) and \( \Delta_{w} \). The parametric calculations give the following perspectives: The magnitude of \( K/K_0 \) becomes smaller when the magnitude of \( \alpha \) increases, namely, the
Table 1 Experimental parameters used

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s$ (kPa)</td>
<td>16.4</td>
</tr>
<tr>
<td>$\varphi$ (deg.)</td>
<td>25.0</td>
</tr>
<tr>
<td>$\phi_e$ (deg.)</td>
<td>50.0</td>
</tr>
<tr>
<td>$G^*$ (MPa)</td>
<td>100.0</td>
</tr>
<tr>
<td>$\Delta_{sp}$ (%)</td>
<td>0.001</td>
</tr>
</tbody>
</table>

*G at shear strain $\gamma = 10^{-4}$ (Yasufuku et al., 2001)
**Calculated values by eq.(9)

\[
\mathbf{f}_s = \rho_m \tan \phi_e
\]

\[
\rho_m = \frac{K \sigma_s}{\gamma}
\]

\[
K = \left( 1 + \frac{\pi \phi_s}{\mu} \right) \frac{\mu}{\tan \phi_s} \tan \phi_e
\]

\[
\beta = \frac{\Delta_{sp}}{\sigma_s}
\]

\[
\lambda_o = \frac{\Delta_{sp}}{\sigma_s} \frac{\mu}{\tan \phi_s}
\]

\[
\lambda_s = \frac{G}{K \sigma_s} \tan \phi_e
\]

\[
\omega = \frac{\mu}{1 + \tan \phi_e}
\]

Fig.4 Outline of the estimated method presented

Then, the pile diameters $D_z$ and $D_{z+z}$ at certain depth $z$ before and after the pile penetration of "S", as shown in Fig.3, respectively, are easily represented by the following relationships:

\[
D_z = \frac{L-z}{L} (D-d) + D \quad D_{z+z} = \frac{L-z+z}{L} (D-d) + D
\]

Based on these two equations, the pressing rate due to pile penetration of "S" is easily derived as

\[
\frac{dr}{r_0} = \frac{D_z - D_{z+z}}{D} \frac{d}{L} = \frac{D(1-d/L)}{(L-z) + \frac{d}{L}} \frac{S}{D}
\]

where, $r_0$ is a radius of the taper shaped pile at depth $z$ and $dr/r_0$ is an increment of pile radius due to the pile penetration. The pressing rate $dr/r_0$ obtained can directly calculate using the above equation.

3.5 Evaluation of penetration factor linked with pressing rate

In order to link the penetration factor $\alpha$ with the pressing rate $dr/r_0$ mentioned above, $R_1$ is first assumed to be equivalent to $r_1$, and also $R_3$ is assumed to represent as a linear relationship. According to this treatment, the penetration factor $\alpha$ is expressed as follows:

\[
\alpha = \frac{R_1}{R_0} = \frac{r_1}{r_0} \frac{1}{\mu} (hr + r_3 + 1)
\]

where, $\mu$ is a correction factor related to soil compressibility and stress relaxation, which is
generally less than 1.0. In this study, the frictional resistance mobilized, when the settlement of taper shaped pile normalized by the diameter of the tip becomes 0.1, is assumed to be the ultimate one. The pressing rate \( \Delta t/\alpha \) is concretely estimated by introducing the ultimate settlement into eq.(14), and then a taper shaped pile skin resistance, reflecting the soil compressibility, shear deformation and strength properties is easily calculated by combining eqs.(8) and (15) with eq.(4). The outline of the model presented here is shown in Fig.4.

4. VERIFICATION BY MODEL TESTS

A series of model loading tests of taper shaped pile with the taper angle \( \theta \) of 0.7 (degs.) were carried out for comparing the predicted skin friction under the various overburden pressures with the experimental one. The details of model load tests have been already reported by Yasufuku et al., (2004). The experimental parameters for the predictions are summarized in table 1, where it is noted that \( \Delta t \), \( \mu \) is estimated by eq.(9) as a function of \( \phi' \), \( G \) and \( \sigma_0 \). Figure 5 compares the experimental ultimate skin frictions at the normalized settlement of 0.1 defined above under four different overburden pressures with the predicted results in terms of various values of \( \alpha \). It is found that, irrespective of the magnitude of the overburden pressure, the predicted results with penetration factor \( \alpha \) in the range of 0.998-0.999 give a good agreement with the experimental results. When considering that the pressing rate \( \Delta t/\alpha \) obtained geometrically is around 0.042, the corresponding correction factor \( \mu \) in eq.(15) automatically becomes in the range from 0.962 to 0.961, which may reflect that the experimental results are influenced by the stress redistribution and relaxation of ground surrounding the pile. Therefore, when the correction factor \( \mu \) is properly evaluated in connection with \( \alpha \), the estimation method proposed here can evaluate the skin friction of the taper typed pile during the vertical penetrations.

5. CONCLUSIONS

A practically useful and rational evaluation method of skin friction of a taper shaped pile in sandy soil ground is discussed based on the geomechanical points of view. The important results obtained are summarized as follows:
1) A simple and rational estimating method for predicting the skin friction of taper shaped type pile was proposed based on the cavity expansion theory. It is characterized that the pressing effect of a taper pile during the penetration is concretely expressed as a result of geometrical considerations of the taper shaped pile, which is linked with soil compressibility derived by an extended cavity expansion theory newly introducing an index, which can evaluates the degree of the pressing effect.
2) The model has only four parameters for predicting the skin friction of the taper shaped pile, which are soil unit weight \( \gamma \), internal friction angle at critical state \( \phi_0 \), shear stiffness \( G \) at 10^-3 strain level and the degree of the pressing effect \( \alpha \), where \( \alpha \) is roughly determined as 0.998-0.999 from an evidence of the predicted results.

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REFERENCES