Point resistance of sheet shaped foundations in sands with consideration of skin friction

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ABSTRACT: In order to calculate bearing capacities of foundations installed in the ground, surcharges due to soil weight has to be known as a boundary stress condition. Equivalent surcharges are conventionally assumed to calculate them. However, the boundary condition seems to influence on the calculation results. In this paper, a formula for the proper surcharge distribution which can take account of an effect of friction at interface between the foundation and the soil is proposed, and then the formula is introduced into the bearing capacity calculations. As a result of the calculations, reductions in the bearing capacities are observed.

1 INTRODUCTION

To calculate bearing capacities of foundations installed under the ground surface by use of classic plasticity theory such as the slip line method and the limit analysis, etc., surcharges due to soil weight has to be known as a boundary stress condition. As a general rule, a product of the unit weight of the soil, \( \gamma \) and depth of the foundation, \( z \) is considered to be the overburden pressures acting above the equivalent free surface. For the ground that carry surcharge, \( q \) at surface, the vertical stress at depth \( z \) is estimated as \( q+\gamma z \). However, it is believed that the vertical stress near foundation is reduced, to be smaller than the conventional overburden pressures, by effect of frictions along foundation skin. This contrast might bring the differences in calculation results of the bearing capacities.

2 OBJECTIVE

This paper tries to obtain a formulation which express proper vertical stress distributions around a sheet shaped foundation by taking account of the friction effect at foundation skin. This formulation will be used as overburden pressure or equivalent surcharge for bearing capacities calculations. The final objective is to show the modified bearing capacities by introducing the proposed surcharges into the slip line analysis.

3 VERTICAL STRESS DISTRIBUTION

3.1 Modeling of ground around foundation

This paper dealt with a sheet shaped foundation installed in the ground which carries the uniform surcharge, \( q \), at the surface. The depth of the foundation is \( z \). The soil around foundation is divided into two zones as shown in Figure 1a. In the first zone, we assumed "reduction zone" where the vertical stress is reduced by the effect of friction along foundation wall. This effect is decreased and will be zero at distance of \( L \). The second zone, "normal zone", is the outer area of the reduction zone. In this area the vertical stress has no reduction and the vertical stress is equal to \( q+\gamma z \).

3.2 Assumptions for this study

In this study, the bearing capacities of sheet shaped foundations are calculated by the slip line method. In order to evaluate the surcharge which can take account of the effect of the skin friction, some assumptions were made. The first assumption is concerned about the vertical stress under the ground at any depth. We assumed that the vertical stress at foundation wall surface is reduced due to skin friction and can be written as \( \alpha(q-\gamma z) \), where \( \alpha \) is reduction factor, which is assumed to be a function of distance from foundation wall surface, \( x \), and depth, \( z \), measured from ground surface. The effect of friction will be decreased linearly when the distance is far away from the foundation wall and reached to zero at a distance of \( L \). Therefore, the vertical stresses at any depth, \( z \), are equal to \( \alpha(q-\gamma z) \) and \( (q-\gamma z) \) at foundation skin surface and at distance of \( L \), respectively, as shown in Figure 1b.
where, $K_0$ is coefficient of earth pressure at rest, and it can be expressed as:

$$K_0 = 1 - \sin \phi$$

(4)

From the study by Ochiai(1977), $\delta$ is assumed to be equal to interparticle friction angle, $\phi_\mu$, and a relationship between $\delta$ and $\phi$ can be expressed as:

$$\sin \delta = \sin \phi_\mu = \frac{\sin \phi}{2 - \sin \phi}$$

(3)

3.3 Derivation of vertical stress distribution

In order to derive the vertical stress distribution, the authors would like to express the derivation step in the form of flow diagram as shown in Figure 2.

First, consider the equilibrium equation in vertical and horizontal direction

Vertical direction:

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} = \gamma$$

(5)

Horizontal direction:

$$\frac{\partial \sigma_h}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

(6)

The second assumption is made on shear stress. Generally, the shear stress under the ground on vertical, $\tau_{xy}$, and horizontal, $\tau_{zx}$ plane are equal to zero because both planes are considered to be principal planes. However, in this study, it is considered that a shear stress is generated in the soil which is located at foundation wall. This shear stress is following the Mohr-Coulomb failure criteria (James, 1994) and has value of $a+K_0\alpha(q+\gamma z)\tan \delta$. Where “a” is adhesion between the foundation surface and the soil; $\delta$ is friction angle between the foundation and the soil and $K_0$ is the coefficient of earth pressure at rest. As mentioned before, at a distance $L$ from foundation, there is no effect from shear stress. Therefore, the shear stress at distance of $L$ is turned to be zero as the consideration in normal case. Thus the shear stress distribution at base level can be illustrated as shown in Figure 3b and the shear stress distribution equation can be expressed by following equation:

$$\tau_{zx} = (a + K_0\alpha(q + \gamma z)\tan \delta) \left(1 - \frac{x}{L}\right)$$

(2)

Figure 2 Flow diagram expresses the vertical stress derivation step used in this study
By differentiating Equations (1) and (2) and substituting them into Equations (5) and (6), two differential equilibrium equations can be obtained as follows:

\[
\begin{align*}
\left(1-\frac{x}{L}\right)\alpha' + (q+z)\frac{\partial \alpha}{\partial z} + \gamma \frac{x}{L} - \frac{K_o}{L}(q+z)\tan \delta \\
+ \left(1-\frac{x}{L}\right)K_o(q+z)\tan \delta \frac{\partial \alpha}{\partial x} = \gamma \\
- \left(1-\frac{x}{L}\right)K_o \tan \delta \frac{\partial (q+z)\frac{\partial \alpha}{\partial x} + \alpha' \gamma}{\partial x} = 0
\end{align*}
\] (7)

By solving these two equations, the following differential equation can be obtained:

\[
\frac{\partial \alpha}{\partial z} = \frac{B + C(q + z) - D\alpha'}{D(q + z)}
\] (9)

where

\[
B = \frac{\alpha - \gamma}{L}
\] (10)

\[
C = \frac{K_o}{L} \tan \delta
\] (11)

\[
D = \left(1-\frac{x}{L}\right) \left[K_o \tan^2 \delta + 1\right]
\] (12)

By integrating Equation (9), the general solution of reduction factor can be derived as:

\[
\alpha = \frac{Bz + Cqz + \frac{1}{2}Cqz^2 + D(\text{const.})}{D(q + z)}
\] (13)

where "const." is a constant of integration which can be determined by a boundary condition that the vertical stress at ground surface \((z = 0)\) is equal to surcharge, \(q\) and \(\alpha\) is equal to 1. Therefore, the constant of integration can be obtained and has a value of \(q\). Finally the specific solution for the reduction factor can be expressed as:

\[
\alpha = \frac{Bz + Cqz + \frac{1}{2}Cqz^2 + Dq}{D(q + z)}
\] (14)

Equation (14) expresses the reduction factor as a function of \(x\), \(z\) and \(L\). However, in this stage, the effective length, \(L\), is still unknown. In order to determine \(L\), another boundary condition where the vertical stress is equal to \(q+\gamma\) at \(x = L\) is applied. Therefore, the distance \(L\) can be expressed as:

\[
L = \frac{a}{L} + \frac{K_o q}{\gamma} \tan \delta + \frac{K_o \gamma}{2} \tan \delta
\] (15)

Finally, the vertical stress distribution at depth \(z\) can be expressed as Equation (1) where the reduction factor, \(\alpha\), and effective length, \(L\), can be calculated by using Equations (14) and (15), respectively.

4 STUDY ON THE VERTICAL STRESS DISTRIBUTION

As mentioned above, in order to obtain the vertical stress distribution, the reduction factor and effective length must be calculated beforehand. As can be seen from Equations (14) and (15), \(\alpha\) and \(L\) depend on unit weight of the ground, surface surcharge and the properties of interface between the soil and the foundation. In this section the effects of these parameters on \(\alpha\) and \(L\) will be studied. Then the typical proposed vertical stress distribution will be shown.

4.1 Effect of soil parameters on reduction factor and effective length

In the Equation (14) with Equations (3), (4) and (15), the \(\alpha\) is a function of \(x\), \(z\), \(\gamma\), \(\alpha\), \(\phi\) and \(q\). The reduction factor will be reduced when the shear stress along foundation skin is increased. In other words, the reduction factor is decreased when the friction angle of the ground increases. However, when considering the characteristic in Equation (14), \(\alpha\) becomes independent of \(x\) for the specific value of \(L\) which is calculated from Equation (15). Therefore, to obtain vertical stress distribution at base level, the reduction factor will be considered to be a constant for any depth of the foundation.

The reduction factor is also independent of cohesion of the ground. Moreover, for the clay case \((\phi = 0)\), there is no reduction in vertical stress.

The effective length, \(L\), indicates the area that the vertical stress has the reduction. Consider Equation (15), the effective length will be increased when the shear stress at the foundation wall or the surface surcharge is increased. However, for highly frictional ground, even though \(\tan \delta\) is increased, the coefficient of earth pressure at rest, \(K_o\) is decreased. From this reason, the effective length tends to be reduced shorter as the friction angle increases. Moreover, as can be seen from Equation (15), the effective length tends to be decreased when the unit weight is increased.
4.2 Reduction factor and effective length for non-surcharge case in sand

This paper focuses mainly on the case where the foundations are installed in sands. For the most cases of sheet shaped foundation, there is no surcharge, \( q \), on the ground surface. Thus, it is important to study the effect on \( \alpha \) and \( L \) values in the cases of no surcharge.

For the sandy ground that has no surcharge at the surface, the reduction factor equation in Equation (14) can be rewritten and expressed as:

\[
\alpha = \frac{\gamma + \varepsilon K_\delta}{K_\delta \tan \delta - \frac{1}{2}} \left[ K_\delta \tan^2 \delta + 1 \right]^{-1}
\]  

(16)

Figure 3 illustrates relationship between reduction factor and friction angle of the ground. From this figure, it can be seen that the reduction factor changes non-linearly from 0.87 and reaches to 1.00 when friction angle changes from 50° to 0°. Therefore, it can be said that the high frictional ground has much more effect on vertical stress compared with low frictional ground and the effect will be vanished for non-frictional ground.

![Figure 3 Variation of reduction factor of the ground, which has no surcharge at surface, against friction angle](image)

For the non-surcharge sandy ground, the effective length equation in Equation (15) can be written as:

\[
L = \frac{K_\delta \varepsilon}{2} \tan \delta
\]  

(17)

As can be seen in the Equation (17), the effective length increases linearly with \( z \). Figure 4 shows variations of \( L \) from ground surface to depth of 20 m. Each line illustrates the effective length in the ground that has different friction angle. This line also shows the interface between reduction zone and normal zone, as mentioned before. To see the effect of friction angle on the effective length clearly, the inclinations of the interface lines, \( \beta \), were plotted against the friction angle as shown in Figure 5.

![Figure 4 Effect of friction angle and depth of foundation on effective length](image)

![Figure 5 Effect of friction angle on the inclination of interface line between reduction zone and normal zone](image)

4.3 Vertical stress distribution for non-surcharge case in sand

The vertical stress distribution within the non surcharge sandy ground can be calculated from Equation (18) which is led from Equation (1).

\[
\sigma_z = \alpha \gamma z \left[ 1 - \frac{x}{L} \right] - \varepsilon \frac{x}{L}
\]  

(18)
The reduction factor and effective length can be calculated as mentioned in Section 4.2.

Figure 6 shows typical vertical stress distribution which is normalized by unit weight of the ground. Note that this figure was calculated by using friction angle of 35°. The reduction of vertical stress near foundation wall and the effective length are relatively small at a shallow depth but the reduction of vertical stress becomes larger and the effective length extends longer at a deeper level.

![Normalized vertical stress distribution](image)

**Figure 6** Normalized vertical stress distribution in non-surge sandy ground which has friction angle 35° at various depth.

5 BEARING CAPACITIES FROM PROPOSED VERTICAL STRESS DISTRIBUTION

5.1 Calculation bearing capacities by slip line method

As previously mentioned, the bearing capacities are calculated by use of the slip line method. The conventional slip line method assumes that the equivalent surcharge applied at equivalent free surface is a constant value, \( q + \gamma z_0 \). By using an additional iteration procedure with the slip line method, bearing capacities which are taken account of the effect of the vertical stress distributions expressed in Equations (1) and/or (18) can be obtained. Note that, when we performed the calculations, the depth of foundation “\( z_0 \)" was used instead of “\( z \)" in all equations.

5.2 Comparison of bearing capacities results

Figure 7 and 8 show examples of calculation results. In these figures, all foundations were installed in the sandy ground having the friction angle 35° and 45°. The solid and hollow dots express the bearing capacities obtained by using the conventional and the proposed vertical stress distribution, respectively. In both cases, the bearing capacities increase linearly with the depth of foundations. It can be seen that when using the proposed method, the bearing capacities become small compared to those obtained by conventional procedure. The gaps tend to increase with increasing the depth of foundation. However, these reductions are somewhat small, and are almost the same value when the depth of foundation is comparatively shallow. From all our calculations (for the ground has friction angle range between 30° to 45°), it appeared that these reductions of bearing capacities are in the range 0.2 to 4.5 %.

With these small reductions, this proposed vertical stress distribution may be neglected from practical point of view.

![Calculated bearing capacity](image)

**Figure 7** Calculated bearing capacities of sheet shaped foundations at various depth (friction angle of the ground = 35°)

![Calculated bearing capacity](image)

**Figure 8** Calculated bearing capacities of sheet shaped foundations at various depth (friction angle of the ground = 45°)

However, it is considered that the reduction is influenced by the degree of the skin friction. In order to confirm the effect of skin friction on the bearing capacities, additional calculations were performed by assuming \( \delta = \varphi \). Figure 9 shows the comparisons of the calculation results with respect to the friction angle of the ground. The circle dots express bearing
capacities calculated by conventional method and the triangle and square dots express the bearing capacities obtained by the skin friction of δ and φ, respectively. To see the reductions clearly, the depth of the foundations was considered to be deep level case (20 m. depth). The unit weight of the soil is 20kN/m³. From this figure, it can be seen that high skin friction results in a large reduction in bearing capacity but the degree of the reductions are still insignificant.

To investigate the validity of this theoretical approach and make clear the effect of skin friction on the bearing capacities, we intend to conduct an experimental study in future.

6 CONCLUSION

In this paper, the vertical stress distribution was derived by considering the effect of skin friction of the foundation. Through a proposed concept, some reductions on vertical stress can be seen near the foundation. By introducing the proposed concept in to the bearing capacity calculations, some reductions in bearing capacities were observed. This reduction indicates that the actual bearing capacities are less than one obtained by using conventional method. However, this observed reduction is in small range from a viewpoint of the practice. Nevertheless, we believe that this concept will provide various applications in practical design such as a stress evaluation of group effect in pile foundations and so on.

REFERENCES