Prediction of Soil Collapse by Lunar Surface Operations in Reduced Gravity Environment

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Abstract. As a basic study for future activities on Moon, influences of the gravity on soil collapse problems such as the bearing capacity and the earth pressure were investigated from the standpoint of plasticity theory. The dimensional analysis was firstly conducted to find the governing parameters for each problem. Then, the slip line method was performed by changing these parameters parametrically. As the results of the calculations, the soil failure mechanisms under the different gravitational conditions were predicted and the influences of the gravity were discussed.

Keywords. Lunar exploration, Reduced gravity, Soil collapse, Bearing capacity, Earth pressure, Slip line method, Dimension analysis

1 Introduction

A long-term vision for space development projects including lunar exploration was officially announced by Japan Aerospace Exploration agency. Physical environment of the lunar surface features some particular aspects such as the absence of an atmosphere, reduced gravity and mechanical peculiarities of the lunar soil, etc.

When we assume future activities, like investigations and constructions on the lunar surface, it is important to estimate soil-machine and/or soil-structure interactions. In this study, we focused on the fact that the gravity of the moon is about one-sixth that of Earth, and theoretically investigated the influences of the reduced gravity on soil behavior. Soil collapse problems such as bearing capacity and earth pressure problems were dealt with in this paper as a basic study.

There are a lot of studies concerning artificial gravitational fields by using centrifuge test apparatus so far. These studies were performed with miniature test apparatus in gravitational acceleration field for the purpose of modeling of soil behavior in actual scale (1.0 g field). This is based on the scaling law of centrifuge modeling, so that stress similarity is achieved at
homologous points by accelerating a model of scale $N$ to $N$ times Earth's gravity. However, as to the reduced gravity, experimental verification of soil behavior in that environment is not easy and has not been achieved so far, because it is difficult to actualize a gravitational reduced field as long as a special test is not used. Therefore, we can not help depending on theoretical analyses or numerical simulations.

In this paper, slip line method (method of stress characteristics) were applied so that stress distributions in failure regions and configurations of slip lines under the desired gravity conditions could be obtained by changing terms of the gravitational acceleration in governing equations.

2 Slip line method

Slip line method allows to provide stress distribution that satisfies both stress equilibrium conditions and the Mohr-Coulomb failure criteria. Combing equations of these conditions under the plane strain condition yields to following partial differential equations and basic characteristics curves known as Kötter's equation;

$$\pm \frac{dp}{dx} + 2(p \tan \phi + c) \frac{d\alpha}{dx} = \frac{\rho \cdot g \cdot \cos(\alpha \pm \mu)}{\cos \phi \cos(\alpha \mp \mu)}$$

$$\frac{dz}{dx} = \frac{\sin 2\alpha \pm \cos \phi}{\sin \phi \cos 2\alpha} = \tan(\alpha \mp \mu)$$

where, $p$ : mean principal stress $= (\sigma_1 + \sigma_3)/2$, $\alpha$ : inclination of the direction of major principal stress from x-axis, $c$ : cohesion, $\phi$ : angle of internal friction, $\rho$ : soil density, $g$ : gravitational acceleration, $\mu : \pi / 4 - \phi / 2$.

Failure regions can be formed by numerical integrations with the equations (1) and (2) in consideration of following four types of boundary problems.

(i) Cauchy problem: when boundary conditions on a non-slip line were known, a stress on a slip line generates from the non-slip line may be found by calculations of the Cauchy problem.

(ii) Riemann problem: when stresses on two slip lines were known, new slip lines may be formed by numerical calculations using finite difference scheme with respect to the equations (1) and (2).

(iii) Problem around the singular point: when a rotation angle of major principal stresses on a singular point was known, the solution around the singular point may be found by dealing with the special case of (ii) with one slip line reduced to a point

(iv) Mixed boundary problem: when a stress on a slip line and a direction of a major principal stress on a non-slip line were known, stress on the non-slip line may be found by combining case of (ii) and (iii).

In practice the above problems are combined in variety of manners according to the problem that we deal with. For instance, the bearing capacity of a smooth strip footing is obtained by solving the Cauchy problem on ground surface, followed by the mixed boundary problem on the edge of the footing base and the Riemann problem within the failure regions, and finally the mixed boundary problem on the footing base to give the required stresses.
3 Dimensional analysis

The slip line method is capable of calculating by individually inputting soil parameters and boundary conditions corresponding to each problem. However, by considering dimensionless parameters which intrinsically govern the soil deformation, it can be possible to make clear the mechanism of soil failures in reduced gravity environment. The dimensionless parameters can be obtained by dimensional analysis based on the Buckingham Pi theorem\(^1\). The basic premise of dimensional analysis is that physical principle must be independent of the choice of units. For example, Newton's law \( F = ma \) must predict the same physical phenomena whether we use different units. In other words, if only the dimensionless parameters were kept constantly, physical phenomena such as stress and shape of the slip line must be similar.

Here, consider two simple examples; bearing capacity of strip footing and earth pressure on vertical wall under the plane strain condition. From the standpoint of plasticity theory, quantities which seem to participate in the failure mechanism were chosen as listed in Table 1. Note that, dimensions in this table were determined so as to correspond with the plane strain condition, e.g., for bearing capacity, though the dimension would be generally expressed as \( ML^{−1}T^{−2} \) (a force acts on an unit area), it is expressed as \( MT^{−2} \) (a force acts on an unit length by getting rid of one dimension of the length in the direction which is perpendicular to the failure plane).

As shown in Table 1, eight quantities are chosen to govern each failure mechanism. Applying dimensional analysis, it is possible to reduce the number of governing parameters by use of dimensionless parameters. As for the bearing capacity problem, following procedure can be conducted.

The bearing capacity \( q_f \) is able to be expressed by a power form of physical quantities as follows:

\[
q_f = k \cdot B^{4_1} \cdot q_0^{4_2} \cdot \rho^{4_3} \cdot g^{4_4} \cdot c^{4_5} \cdot a^{4_6}
\]  

Substituting each dimension shown in Table 1 into each physical quantity yields the following expression.

**Table 1 Physical quantities and its dimensions**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Dimension*)</th>
<th>Quantity</th>
<th>Symbol</th>
<th>Dimension*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing capacity</td>
<td>( q_f )</td>
<td>( MT^{−2} )</td>
<td>Thrust force</td>
<td>( P )</td>
<td>( MLT^{−2} )</td>
</tr>
<tr>
<td>Breadth of footing</td>
<td>( B )</td>
<td>( L )</td>
<td>Height of wall</td>
<td>( H )</td>
<td>( L )</td>
</tr>
<tr>
<td>Surcharge</td>
<td>( q_0 )</td>
<td>( MT^{−2} )</td>
<td>Surcharge</td>
<td>( q_0 )</td>
<td>( MT^{−2} )</td>
</tr>
<tr>
<td>Soil density</td>
<td>( \rho )</td>
<td>( MT^{−2} )</td>
<td>Soil density</td>
<td>( \rho )</td>
<td>( MT^{−2} )</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>( g )</td>
<td>( LT^{−2} )</td>
<td>Gravitational acceleration</td>
<td>( g )</td>
<td>( LT^{−2} )</td>
</tr>
<tr>
<td>Cohesion</td>
<td>( c )</td>
<td>( MT^{−2} )</td>
<td>Cohesion</td>
<td>( c )</td>
<td>( MT^{−2} )</td>
</tr>
<tr>
<td>Adhesion **)</td>
<td>( a )</td>
<td>( MT^{−2} )</td>
<td>Adhesion **)</td>
<td>( a )</td>
<td>( MT^{−2} )</td>
</tr>
<tr>
<td>Internal friction angle ***)</td>
<td>( \phi )</td>
<td>–</td>
<td>Internal friction angle ***)</td>
<td>( \phi )</td>
<td>–</td>
</tr>
<tr>
<td>Friction angle ***)</td>
<td>( \delta )</td>
<td>–</td>
<td>Friction angle ***)</td>
<td>( \delta )</td>
<td>–</td>
</tr>
</tbody>
</table>

\(^1\)Dimensions were determined so as to correspond with the plane strain condition.

\(^{**}\) Adhesion between soil and footing base or wall.

\(^{***}\) Friction angle between soil and footing base or wall.
\[ MT^{-2} = k \cdot (L)^{\lambda_1} (ML^{-2})^{\lambda_2} (L^{-2})^{\lambda_3} \left( MT^{-2} \right)^{\lambda_4} \left( MT^{-2} \right)^{\lambda_5} \]

By comparing the powers between the right and the left side of the equation (4) with respect to the basic dimensions, \( M, L, \text{ and } T \), relations for the powers can be obtained as follows:

For \( M \):
\[ 1 = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 \]

For \( L \):
\[ 0 = \lambda_4 - 2\lambda_3 + \lambda_4 \]

For \( T \):
\[ -2 = -2\lambda_2 - 2\lambda_4 - 2\lambda_5 - 2\lambda_6 \]

From the equation (5), (6) and (7), it can be obtained the relation, namely, \( \lambda_1 = \lambda_3 = \lambda_4 \), \( \lambda_5 = 1 - \lambda_2 - \lambda_3 - \lambda_6 \). Restoring these relations into the equation (3) yields the following expression.

\[ q_f = k \cdot c \cdot \left( \frac{B}{c} \right)^{\lambda_3} \left( \frac{q_0}{c} \right)^{\lambda_2} \left( \frac{\rho \cdot g}{c} \right)^{\lambda_4} \left( \frac{a}{c} \right)^{\lambda_6} \]

Consequently, the bearing capacity normalized by cohesion can be written by the following functional form.

\[ \frac{q_f}{c} = F \left( \frac{\rho \cdot g \cdot B}{c}, \frac{q_0}{c}, \frac{a}{c}, \phi, \delta \right) \]

As to the earth pressure problem, the normalized force on vertical retaining wall can be express as:

\[ \frac{P}{c \cdot H} = F \left( \frac{\rho \cdot g \cdot H}{c}, \frac{q_0}{c}, \frac{a}{c}, \phi, \delta \right) \]

Therefore, by changing the dimensionless parameters in the above functions, the solutions which are independent of scales of the field could be obtained.

4 Calculation results and discussions

Figure 1 depicts slip line nets obtained by changing the term of the gravitational acceleration as \( 1g \) and \( 1/6g \). The calculations were performed for each soil collapse problems by considering the interfaces between soils and structures as completely smooth (\( \delta = 0, a = 0 \)) or completely rough (\( \delta = \phi, a = c \)). Soil parameters used in the calculations are given in Table 2. These are the values that Carrier, W. D. III et al.\(^2\) had been reported in the Lunar Sourcebook. They tested returned lunar soil samples to investigate the physical and mechanical properties.

Furthermore, Figure 2 represents ratios of the collapse load and the size of failure region in \( 1/6g \) condition to those of the cases in \( 1g \) condition. The size of failure region is compared with the length, \( L \) between the origin and the point that outermost slip line reaches the ground surface.

As shown in the Figure 1 and 2, the gravity has a significant influence on the collapse loads especially for the footings and the passive earth pressures. As to the active earth pressure problem, it seems that the influence of gravity is comparatively small. Anyhow, it can be said that reduced gravity environment gives safe side solutions in all cases because the ratio of collapse loads never fall below 16.7 % (=1/6). Meanwhile, it can be seen that the failure regions becomes larger when the gravity is reduced. The reason seems to be that the downward stress component by the gravity is reduced and it affects on the rotations of the major principal stresses.
Figure 1  Slip line nets describe by slip line method ( $c = 0.9$ kN/m$^2$, $\phi = 46$ degree)

Table 2  Soil parameters of lunar soil samples$^{2}$

<table>
<thead>
<tr>
<th>Depth range (cm)</th>
<th>Void ratio</th>
<th>Average bulk density (g/cm$^3$)</th>
<th>$c$ (kN/m$^2$)</th>
<th>$\phi$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>Range</td>
</tr>
<tr>
<td>0 - 15</td>
<td>1.07 ± 0.07</td>
<td>1.50</td>
<td>0.52</td>
<td>0.44 - 0.62</td>
</tr>
<tr>
<td>0 - 30</td>
<td>0.96 ± 0.07</td>
<td>1.58</td>
<td>0.90</td>
<td>0.74 - 1.10</td>
</tr>
<tr>
<td>30 - 60</td>
<td>0.78 ± 0.07</td>
<td>1.74</td>
<td>3.00</td>
<td>2.40 - 3.80</td>
</tr>
</tbody>
</table>
Concerning the smooth and rough case, the collapse loads can be considered as the functions of two dimensionless parameters, $\rho gB/c$ and $q_0/c$. Figure 3 represents solution surfaces for the cases of $\phi = 46$ degree. These figures show that the effect of the gravity is prone to appear when the terms of the surcharge is comparatively small, while the effect of the surcharge is subjected when the gravity is small. That is to say, considerations of the influence of gravity play an important role where the surcharge is small, e.g., shallow footings, shallow excavations, etc.

When the surcharge is zero, influence of $\rho gB/c$ on collapse loads are shown in Figure 4. It can be seen from these figures, the collapse loads dramatically changes at $\rho gB/c = 1$ or more in all cases. Note that the not only the gravity but also bulk density of soil, breadth of the footing, height of the wall and cohesion influence as well on the soil collapse because the governing parameter for gravitational term consists of these individual parameters. In the meantime, from

Figure 2 Comparisons of collapse loads and failure regions

Figure 3 Solution surfaces against the governing parameters ($\phi = 46$ degree)
Figure 4 Influences of the gravitational governing parameter on collapse loads

Figure 5 Variations of outermost slip lines under the different gravity conditions

the Figure 4(c), influence of gravity seems to become remarkable as $\phi$ decreases regarding the active earth pressure problem.

As mentioned above, the shape of the failure region will change by the magnitude of gravity. Figure 5 represents the variations of the outermost slip line under the different gravity conditions. As can be seen from the Figure 5(a) and (b), the failure region for footings becomes smaller as $\rho gB / c$ increases. In the case of the earth pressure problem, it was made clear that the gravity affects on the passive case for rough walls, whereas it doesn't affect on the active case whether it is smooth or rough.
The collapse loads are generally estimated by the principle of superposition with factors or coefficients as follows:

For bearing capacity problem: 
\[ q_f = N_c q + N_q + \frac{1}{2} \gamma B N_\gamma \]  
(11)

For earth pressure problem: 
\[ P = K_c H + K_q h + \frac{1}{2} \gamma H^2 K_\gamma \]  
(12)

where, \( N_c, N_q, N_\gamma \): bearing capacity factors with respect to cohesion, surcharge and self weight, respectively, \( K_c, K_q, K_\gamma \): coefficients of earth pressure with respect to cohesion, surcharge and self weight, respectively.

It is considered that the values of \( N_q \) and \( N_\gamma \) may change with the gravity, because it was made clear that the collapse loads fluctuate with the soil governing parameters, \( \rho g B / c \) and \( q_0 / c \) as shown in the Figure 3. Concerning a bearing capacity problem in case of a smooth footing with no surcharge, variations of the \( N_\gamma \) value can be plotted as Figure 6. From this figure, it can be considered that the usual superposition is not applicable any longer, and the \( N_\gamma \) value should be changed according to the proper \( \rho g B / c \) if the superposition is applied. However, in the situation that the surcharge is applied as well, the \( N_\gamma \) value itself is also affected by the variations of \( q_0 / c \). Therefore, it is necessary to execute the calculation scheme of the slip line method to directly estimate the collapse load by individually inputting each soil parameter and boundary condition.

5 Conclusions

In this paper, as a basic study for future activities on Moon, influences of the gravity on soil collapse such as the bearing capacity and the earth pressure problem were investigated from the standpoint of plasticity theory. The dimensional analysis was firstly conducted to find the governing parameters for each problem. Then, the slip line method was applied by changing these parameters parametrically and the influences of the gravity were examined. The failure mechanisms under \( 1/6 g \) condition were predicted by the slip line method and compared with those of \( 1 g \) condition. As the results of the parametric studies, it was made clear that the gravity...
has a significant influence on the collapse loads especially shallow footings and passive earth pressures. The effect of gravity on collapse loads is prone to appear when the surcharge is comparatively small. However, the solutions in reduced gravity environment are always given as safe side. Furthermore, the failure regions for footings and passive earth pressures on rough walls become smaller as the gravity increases, while the regions for earth pressure on smooth walls and active cases dose not change with the gravity.

References