Reliability of Vertical Bearing Resistance of Bored Friction Piles

by

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Abstract

This paper describes a method for evaluating the reliability of the vertical bearing resistance of bored friction piles, in which a semi-empirical expression using SPT-N value is focused on. Two uncertainties, which are spatial distribution of SPT-N value and resistance coefficients, are quantitatively evaluated by statistical analyses of various soil investigations and loading test results. Based on these evaluations, a performance factor for the Limit State Design is proposed for the design of bored friction piles. The performance factor is also applied to a bridge foundation design.

Keywords: Bearing resistance, SPT-N value, Bored friction pile, Reliability, Uncertainty, In-situ loading test, Spatial distribution, Limit State Design, Performance factor

1. Introduction

Pile foundations are classified into two major categories according to the mechanism of load transfer to the ground, which are end bearing pile and friction pile1). End bearing pile has been conventionally used as a basic type of pile foundation in Japan, but friction pile may be more reasonable and economical due to the scale of structures and the soil conditions. Bearing resistance of friction pile mainly depends on the frictional one along pile shaft length, resulting in a considerable variation. It is, therefore, very important to evaluate the bearing resistance in the light of reliability. On the other hand, design method for pile foundation is being improved from the Working Stress Design to the Limit State Design (LSD)2,3,4) in which a reliability-based analysis is indispensable.

In this paper, a method for evaluating the reliability of bearing resistance of bored friction pile is presented, when semi-empirical expressions using N-value from the Standard Penetration Test (SPT) are used for calculation of the bearing resistance of the pile. The

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method is based on in-situ loading test results and the consideration of spatial distribution of N-values by a statistical analysis. A performance factor for the Limit State Design is then presented by using the proposed method. Finally, the performance factor is applied to a design of actual bridge foundation.

2. Uncertainties in the Bearing Resistance Expression

2.1 Semi-empirical bearing resistance expression

The ultimate bearing resistance \( R_u \) is generally expressed as a sum of pile tip and shift resistances.

\[
R_u = R_p + R_f = q_d A_p + U \sum_{i=1}^{M} (l_i f_i)
\]  

(1)

in which \( R_p \) = soil resistance at pile tip, \( R_f \) = soil resistance around pile shaft, \( q_d \) = unit tip resistance, \( f_i \) = unit shaft resistance in the \( i \)-th layer of the ground penetrated by the pile, \( A_p \) = area of pile tip, \( U \) = perimeter of pile, \( l_i \) = thickness of the \( i \)-th layer, and \( M \) = the number of soil layers pile penetrates. If the values \( q_d \) and \( f_i \) in Eq. (1) are properly determined, the ultimate bearing resistance \( R_u \) is calculated properly.

A semi-empirical expression for the resistance based on SPT-N value has been widely used for the design of pile foundation in Japan. Eq. (1) is then rewritten as\(^1\):

\[
R_u = \alpha_p N_p A_p + U \sum_{i=1}^{M} (\alpha_f l_i N_i)
\]  

(2)

where \( \alpha_p \) = tip resistance coefficient, \( \alpha_f \) = shaft resistance coefficient in the \( i \)-th layer, \( N_p \) = SPT-N value near the pile tip, and \( N_i \) = SPT-N value in the \( i \)-th layer.

In order to estimate the bearing resistance properly by Eq. (2), the quantitative evaluations of SPT-N value and the resistance coefficients \( \alpha_p \) and \( \alpha_f \) should be taken into account\(^5\).

2.2 SPT-N value

The soil properties, such as SPT-N values, exhibit considerable variation, which are usually caused by the following three type of uncertainties\(^6\):

Type I: natural heterogeneity or the in-situ variability of the soil,

Type II: limited availability of information about subsurface conditions, and

Type III: measurement errors due to human factors.

Fujita et al.\(^7\) has investigated the difference of SPT-N values related to the distance between two boring points by using several boring data conducted in Japan. According to the results of this study, the measurement errors due to human factors are negligible in the estimation of SPT-N value, so that it was concluded that the uncertainty of SPT-N value is mainly caused by its spatial distribution due to the natural heterogeneity of the soil.
2.3 Resistance coefficients

According to the Japanese Specifications for Substructures\(^1\), the ultimate bearing resistance of piles were estimated by Eq. (2) with SPT-N values based on in-situ loading test data\(^8\). The total number of loading tests of bored pile was 32, including 16 cases of end bearing piles and 16 cases of friction piles. The ultimate bearing resistance, \(R_{u10}\), was defined as the pile head load at a settlement level of 10% of the pile diameter\(^8,9,10\). Based on these results, Table 1 shows the first and second order statistics of the resistance coefficient, \(\alpha_p\) and \(\alpha_{fi}\), and the resistance ratio, \(P_F\) and \(P_u\), which are defined as follows:

\[
\alpha_{fi} = \frac{f_{im}}{N_i} \quad (3a)
\]

\[
\alpha_p = \frac{q_{dm}}{N_p} \quad (3b)
\]

and

\[
P_F = \frac{R_{Fm}}{R_{Fc}} \quad (4a)
\]

\[
P_u = \frac{R_{um}}{R_{uc}} \quad (4b)
\]

where subscripts \(m\) and \(c\) stand for measured and calculated values, respectively. The variation of the resistance coefficients ranges from 54% to 67% in terms of the coefficient of variation (COV). According to the \(\chi^2\) statistical test with the significance level at 5%, the distributions of the coefficient resemble either a normal distribution or a log-normal distribution. The mean-values of the resistance ratios, \(P_F\) and \(P_u\), are close to one, so that it may be considered that the bearing resistance expression by means of the resistance coefficients, \(\alpha_p\) and \(\alpha_{fi}\), could estimate the measured values without bias. However, the COV of \(P_F\) of 36%, and the COV of \(P_u\) of 30% show large variations. In order to improve the reliability of bearing resistance, it is most effective to carry out the in-situ loading tests at each individual site, and then to estimate bearing resistance with these results.

Table 1 The first and second order statistics of the resistance coefficients and the resistance ratios

<table>
<thead>
<tr>
<th>sample size</th>
<th>Mean-value</th>
<th>Standard deviation ((\sigma))</th>
<th>Coefficient of variation (COV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resistance coefficient</strong></td>
<td>(\alpha_t) (sandy)</td>
<td>40</td>
<td>0.412</td>
</tr>
<tr>
<td></td>
<td>(\alpha_t) (clayey)</td>
<td>35</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>(\alpha_p)</td>
<td>16</td>
<td>10.6</td>
</tr>
<tr>
<td><strong>Resistance ratio</strong></td>
<td>(P_F)</td>
<td>16</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(P_u)</td>
<td>16</td>
<td>0.991</td>
</tr>
</tbody>
</table>
3. Estimation of Spatial Distribution of $STP-N$ Value

3.1 Auto-correlation function of $SPT-N$ value

Uncertainties in soil properties have conventionally been expressed in terms of mean-value and variance as random variables\textsuperscript{11,12,13,14}. However, this is not sufficient to describe the spatial distribution of soil properties. The spatial distribution of soil property is expressed by introducing an auto-correlation coefficient in addition to the mean-value and the variance\textsuperscript{8}. The concept of a random field is based on the presumption of a population. When a site is specified, there exists a sample field. It is, however, impossible to determine this sample field completely, and the soil properties at points excluding the sample points must be estimated. Such estimation introduces errors resulting from spatial distribution of the soil properties. Here in this paper, Kriging technique is used for the estimation of the sample field\textsuperscript{15,16}.

Kriging technique was used by Christakos\textsuperscript{17} and Honjo et al.\textsuperscript{18} for estimating the plane distribution of settlement, and by Honjo and Matsunaga\textsuperscript{19} for compaction control of soil in the geotechnical engineering. Furthermore, Suzuki and Ishii\textsuperscript{20} have tried to apply this method in the stochastic finite element method and have confirmed its effectiveness. In Kriging technique a estimator is generally expressed by a weighted linear sum of sample values considering the unbiasedness of estimator and minimization of estimation error. Supposeing that an estimator $\hat{Z}(x)$ is expressed by the linear sum of the sample value, $z(x_j)$, obtained at the sample points $x_j (j = 1, 2, \ldots, m)$ of $STP-N$ value, the estimator is obtained by:

$$\hat{Z}(x) = \sum_{j=1}^{m} \lambda_j z(x_j)$$  \hspace{1cm} (5)

in which $\lambda_j$ = the weights applied to the $j$-th data. They are determined by using Lagrange multiplier method considering the following two conditions.

$$E[Z(x) - \hat{Z}(x)] = 0$$ : Unbiasness of estimator, and

$$\sigma_E^2 = \min E \left\{ \left( Z(x) - \hat{Z}(x) \right)^2 \right\}$$ : Minimum variance of estimation error\textsuperscript{(7)}

Then, the auto-correlation function, $\rho(x)$, is expressed by:

$$\rho(\Delta x) = C(\Delta x)/C(0)$$ \hspace{1cm} (8a)

in which

$$C(\Delta x) = E[Z(x + \Delta x)Z(x)] - E[Z(x)]^2$$ : covariance \hspace{1cm} (8b)

$$C(0) = E[Z(x)^2] - E[Z(x)]^2$$ : variance \hspace{1cm} (8c)

where $x$ = position coordinates, $\Delta x$ = distance between two sample points, $Z(\cdot)$ = random variable of soil properties, and $E(\cdot)$ = expected value. The mean-value and the variance can
be easily obtained, but it is often difficult to estimate the auto-correlation function because of the limitation in the number of samples. Following one-dimensional auto-correlation model is used in this research\textsuperscript{14,21}.

\[ \rho(\Delta x) = \exp(-\Delta x/A) \]  

(9)

in which \( A \) = correlation parameter indicating the degree of reduction of correlation characteristics.

3.2 Correlation parameters for \textit{SPT-N} value

The characteristics of the parameters of auto-correlation functions for both horizontal (\( z \)-direction) and vertical (\( z \)-direction) directions are investigated separately on the basis of the sample data of \textit{SPT-N} value, where the parameter for \( z \)-direction is indicated as \( A \), whereas for \( z \)-direction \( B \). Fig. 1(a) shows the relationships between the average value of the spacing \( L \) in each adjacent two sample points and the correlation parameter \( A \), which is obtained using the sample data of \textit{SPT-N} value in each soil layer at the same depth. As realized from Fig. 1(a), the value of \( A \) decreases exponentially when the spacing \( L \) becomes small. This tendency is independent of the soil type and is expressed with the equation:

\[ A = \exp(2.97 + 0.00736L) \]  

(10)

The parameter \( A \) should be independent of the spacing \( L \). In order to obtain the unique value of \( A \), the sample points spacing should be small enough so that the relationship is not affected by sampling. Now, supposing \( L = 0 \), then value for \( A \) of approximately 15m is obtained. It is appropriate to consider it as the real value of the correlation parameter of \textit{SPT-N} value in horizontal direction.

The relationship between the sample points spacing \( L \) and parameter \( B \) in the \( z \)-direction are shown in Fig. 1(b). \( B \) is not related to the sample point spacing and is a unique value for that ground. The variation of the \( B \) value due to the soil type is also not clear and is

![Fig. 1 Relation between correlation parameter of \textit{SPT-N} value and sample points spacing in horizontal and vertical direction.](image-url)
ranged from 1m to 5m. The value of $B$ is less than one-third of the value of $A$ (=15m) for the SPT-$N$ value, which is of the same tendency as other soil properties.

3.3 Spatial distribution of SPT-$N$ value due to Kriging technique

The spatial distribution of SPT-$N$ value estimated by using Kriging technique is compared with the measured value in order to verify the effectiveness of Kriging technique. The soil profile at investigation site is shown in Fig. 2, which is ground composed of diluvial clay (Dc layer), sand (Ds layer), and tuff sandy soil (Dt layer). The soil investigation is done in 4 steps (designated as STEP-1 to STEP-4) at the site. Here the SPT-$N$ values at the Dt layer in STEP-3 and 4 are estimated from the investigated values in STEP-2 and 3, respectively.

The estimation method of parameter $A$ in horizontal direction of the Dt layer is indicated in Fig. 3. As shown in Fig. 1(a), parameter $A$ is a function of the spacing $\bar{L}$ and decreases as $\bar{L}$ becomes small. The value of $A$ at $\bar{L} = 0$ is obtained using the least square method with known value of the parameters. As shown in Fig. 3, the results from STEP-1 and 2 are used for obtaining the value of $A$ for STEP-2 while those from STEP-1, 2 and 3 are

![Fig. 2 Soil profile.](image)

![Fig. 3 Decision of correlation parameter A.](image)
Table 2 The analysis conditions of N-values for Dt layer

<table>
<thead>
<tr>
<th>STEP</th>
<th>Sample size</th>
<th>Mean-value (N)</th>
<th>COV (V_N)</th>
<th>Correlation parameter (m) A</th>
<th>Correlation Parameter (m) B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>136</td>
<td>19.8</td>
<td>0.462</td>
<td>32</td>
<td>3.7</td>
</tr>
<tr>
<td>3</td>
<td>261</td>
<td>22.0</td>
<td>0.415</td>
<td>18</td>
<td>4.5</td>
</tr>
</tbody>
</table>

used for that of STEP-3. Thus, the values of A = 32m and 18m are obtained for STEP-2 and 3, respectively.

The auto-correlation function is assumed to be the sum of both the effects in horizontal and vertical direction:

\[
\rho(\Delta x, \Delta z) = \exp \left[ - \left( \frac{\Delta x}{A} \right)^2 + \left( \frac{\Delta z}{B} \right)^2 \right]^{1/2}
\]  

(11)

in which \( \Delta x \) and \( \Delta z \) = the distances between two sample points in the horizontal direction and vertical direction, respectively, and \( A \) and \( B \) = correlation parameters. The analyses conditions of SPT-N value in the Dt layer are listed in Table 2. The values of \( N \) and \( V_N \) are obtained using investigated SPT-N value in STEP-2 and 3. The parameter \( A \) is determined from Fig. 3 while \( B \) is the parameter in the vertical direction obtained by Eq. (9) in STEP-2 and 3, respectively.

Both results of estimation by Kriging technique and the measured values are shown in Fig. 4, in which the estimated SPT-N value at each point in the Dt layer is plotted at the stations. The estimated value (mean-value), and this with ± estimation error (standard deviation), are plotted in the figure. The estimation values approximately agree with the measured values, but the estimation errors are not reduced to zero. It is considered that this is caused by the variation associated with the distribution of N value in vertical direction.

Fig. 4 Estimation of spatial distribution of N-values (Dt layer).
 Needless to say, the estimation errors are largest at the middle point between two sample points. Thus it is concluded that the spatial distribution of SPT-N value may be estimated in the range of mean-value ± standard deviation (1σ) by Kriging technique.

3.4 Simplified expression

In order to propose a performance factor with consideration of the spatial distribution of SPT-N value, a simplified expression is formulated\(^5\). The correlation in the horizontal direction is relatively strong, so that the estimated value is approximated by connecting the measured values between two sample points with a straight line. It may be considered that the COV of the estimated SPT-N value reaches to that of the \( N \) value in that layer when the distance between two sample points is long enough. Fig. 5 shows the relationship between \( \frac{V_{Nik}}{V_{Ni}} \) and \( \frac{\lambda}{A} \) based on the result from Kriging technique, in which \( V_{Nik} \) is the COV of the SPT-N value at the estimation point \( k \) and \( V_{Ni} \) is the COV of the SPT-N value in \( i \)-th layer while \( \lambda \) is a distance between a sample point and an estimation point. There is a linear relation between \( \frac{V_{Nik}}{V_{Ni}} \) and \( \frac{\lambda}{A} \):

\[
\frac{V_{Nik}}{V_{Ni}} = 0.331 + 0.264 \frac{\lambda}{A}
\]  

(12)

It is noted that this is a unique relation for different soil types. When the value of \( \frac{\lambda}{A} \) is equal to 2.5, \( \frac{V_{Nik}}{V_{Ni}} \) is approximately equal to 1.0, so that \( V_{Nik} \) may be the same value as the SPT-N value at the layer for the condition of \( A = 15m \) with \( \lambda = 40m \). Therefore, the estimated value and the estimation error of the spatial distribution of SPT-N value are easily obtained using this proposed simplified expression with the idea of estimated value described above.

For the case of the spatial distribution of SPT-N value in STEP-2 at the Dt layer as shown in Fig. 4, the estimated values by the Kriging technique are compared with the results using the proposed simplified expression in Fig. 6. Both estimated value and estimation error show a good agreement with each other. It means that the proposed simplified expression for the estimation method may have satisfactory accuracy.

![Fig. 5 Relation between COV ratio and normalized distance.](image-url)
4. Evaluation of the Resistance Coefficients Based on Loading Tests

4.1 Probability model of bearing resistance

The dominant uncertainties in Eq. (2) are possibly attributed to the SPT-N value and the resistance coefficients $\alpha_p$ and $\alpha_{fi}$. The values $A_i, U$ and $l_i$ in Eq. (2) are considered to be constant values, while the resistance coefficients $\alpha_p$ and $\alpha_{fi}$ and SPT-N values, $N_p$ and $N_i$, are random variables.

Supposing that these variables are statistically independent, the estimated mean-value of $R_u$ and its variance $\sigma_R^2$ can be derived from Eq. (2) as follows:

$$\overline{R}_u = \bar{\alpha}_p \bar{N}_p A_p + U \sum_{i=1}^{M} (\bar{\alpha}_{fi} l_i \bar{N}_i)$$ (13)

$$\sigma_R^2 = \sigma_p^2 A_p^2 + U^2 \sum_{i=1}^{M} (l_i^2 \sigma_{fi}^2)$$ (14)

in which $\sigma_p^2 = \bar{\alpha}_p^2 \sigma_{N_p}^2 + \sigma_{\alpha p}^2 \bar{N}_p^2$ and $\sigma_{fi}^2 = \bar{\alpha}_{fi}^2 \sigma_{N_i}^2 + \sigma_{\alpha fi}^2 \bar{N}_i^2$

where $\bar{\alpha}_p$ and $\sigma_{\alpha p}^2$ = mean-value and the variance for the resistance coefficient $\alpha_p$ at the pile tip, $\bar{N}_p$ and $\sigma_{N_p}^2$ = estimated value and the variance of estimation error for the SPT-N value at the pile tip, $\bar{\alpha}_{fi}$ and $\sigma_{\alpha fi}^2$ = mean-value and the variance for the resistance coefficient $\alpha_{fi}$ at the pile shaft in $i$-th layer, $\bar{N}_i$ and $\sigma_{N_i}^2$ = estimated value and the variance of estimation error for the SPT-N value at the pile shaft in the $i$-th layer. Furthermore, the COV of the ultimate bearing resistance $V_R$ is obtained by:

$$V_R = \frac{\sqrt{\sigma_R^2}}{\overline{R}_u}$$ (15)

The first and second order of the resistance coefficients $\alpha_p$ and $\alpha_{fi}$ in the Japanese Specifications for Substructures have been given in Table 1. Since the values of COV in
Table 1 are derived from wide variety of soils in Japan, they show a wide range of variation from 0.5 to 0.7. Therefore, when a pile loading test is conducted at a site and also the results from the test are reflected in the evaluation of bearing resistance of the pile, it is expected that the uncertainty in the resistance coefficient could be reduced.

4.2 Variation of resistance coefficients

If the variation of resistance coefficients are known, it could be easy to evaluate the bearing resistance of a pile with Eq. (13) and (14). Usually, there is only one loading test carried out (singular loading test) at one site. Although the mean-value of the resistance coefficients can be obtained, $V_{ai}$ hardly can be known. Moreover, even two or more loading test have been done at one site (plural loading tests), it still has the chance that $V_{ai}$ is unclear. Thus, it is necessary to find a way to estimate $V_{ai}$ from $\bar{\sigma}_{fi}$, because $\bar{\sigma}_{fi}$ is easily obtained as long as loading test is carried out. Now, the data from many plural loading tests are used for this purpose.

Fig. 7 shows that $V_{ai}$ tends to increase alone with $\bar{\sigma}_{fi}$ irrespective of the type of soils. It is also can be seen from Fig. 7 that the data points scatters much more when $\bar{\sigma}_{fi}$ increases, therefore, to estimate $V_{ai}$ from $\bar{\sigma}_{fi}$, a method called regression with non-constant variation is used. In this method the datum points in region of small variance should have more “weight” than those in regions of large variance. Hence, the regression equation by assigning weight to be inversely proportional to the non-constant variance is obtained \(^2\), when the standard deviation corresponding to this estimation is 0.059 $\bar{\sigma}_{fi}$.

$$V_{ai} = 0.158\bar{\sigma}_{fi}$$  \hspace{1cm} (16a)

From Eq. (16a), the Eq. (16b) is obtained to estimate $V_{ai}$ from $\bar{\sigma}_{fi}$ in case that $V_{ai}$ is not directly obtained in in-site loading test.

$$V_{ai} = 0.3\bar{\sigma}_{fi}$$  \hspace{1cm} (16b)

where the probability of exceedance of $V_{ai}$ is 1%.
4.3 Accuracy of the proposed model

4.3.1 In case of plural loading tests

Five piles had been tested. P12, P31, P56 are in A-region and P6 and A2 are in B-region. The bearing resistance of any of these piles will be estimated by using resistance coefficients among them and compared with its measured value. Before discussion some definitions should be given. CASE-1 is to estimate the bearing resistance of a pile using resistance coefficients from other test piles. CASE-2 is to estimate the bearing resistance of a pile using resistance coefficients from its own loading test and is called "own pile estimation". Because the number of pile used to evaluate other piles is only one in this case, the relation \( V_{ni} = 0.3\alpha_f t_i \) in Eq. (16b) is used. Also in this case, because the variation of tip resistance coefficient is unknown from test so the values in Table 1 is adopted in estimation. Unlike "own pile estimation", the "other pile estimation" is the one which the resistance coefficients are obtained from other test pile, in which the number of test pile is one, too. Apparently, the most reliable estimation is in an "own pile estimation" or CASE-2. The CASE-3 is to estimate using the resistance coefficients based on current Specifications in Table 1. Values of mean-value, \( \overline{R}_u \), and its COV, \( V_R \), which are calculated by Eqs. (13), (14) and (15), are given in Table 3. To compare these estimation results with measured ones in terms of estimation accuracy, the parameter \( p \) called error probability is introduced as follows:

\[
p = \int_{-\infty}^{R_{u10}} \frac{1}{\sqrt{2\pi\sigma_R}} \exp \left\{ \frac{1}{2} \left( \frac{x - \overline{R}_u}{\sigma_R} \right)^2 \right\} \, dx \tag{17}
\]

\( p \) is the probability that the estimated bearing resistance is less than the measured one, \( R_{u10} \). That is \( p = P(R_u \leq R_{u10}) \). Being the distribution \( N(\overline{R}_u, \sigma_R) \), when \( p = 0.5 \) it follows that \( \overline{R}_u = R_{u10} \), which means that the more \( p \) closes to 0.5 the better the estimated bearing resistance closes to the measured bearing resistance, \( R_{u10} \). The first and second order statistics of error probabilities of five test piles related to these three CASE's are shown in Table 4. The probability density functions of these three CASE's are given in Fig. 8. In Table 4, the error probability \( p \) of CASE-1 and CASE-2 are 0.45 and 0.536 on

| Table 3 Estimated values (\( \overline{R}_u \)) and the estimation errors (\( \sigma_R \)) |
|-----------------|---|---|---|---|---|
|                | A-region |       | B-region |       |
|                | P12 | P31 | P56 | P6 | A2 |
| CASE-1 \( \overline{R}_u \) | 6.655 | 6.986 | 7.134 | 6.451 | 6.495 |
| \( \sigma_R \) | 1.353 | 1.346 | 0.929 | 1.256 | 1.121 |
| CASE-2 \( \overline{R}_u \) | 6.476 | 6.629 | 6.935 | 6.194 | 6.278 |
| \( \sigma_R \) | 1.778 | 1.483 | 1.811 | 1.651 | 1.160 |
| CASE-3 \( \overline{R}_u \) | 8.540 | 4.031 | 6.169 | 7.037 | 5.062 |
| \( \sigma_R \) | 2.624 | 1.500 | 2.321 | 2.377 | 1.581 |
average, respectively, while \( p \) of CASE–3 is 0.604, which indicates that good estimation for bearing resistance are obtained excepting CASE–3, and the former two are of the same level of estimation accuracy. Furthermore the standard deviations in former two cases are 0.047 and 0.04, respectively, and they are small compared to 0.304 of CASE–3. It is concluded that we can get more reliable estimation for bearing resistance in CASE–1 and CASE–2 by using the proposed method.

### 4.3.2 In case of singular loading test

CASE–4 is the estimation of bearing resistance based on singular loading test result, which is composed of both “own pile estimation” and “other pile estimation” of these five piles. The estimated value \( \bar{R}_w \) and the estimation error \( \sigma_R \) of the test piles are given in Table 5. The values shown by bold line correspond to the “own pile estimation”. Others are “other pile estimation”. In CASE–4 the Eq. (16b) is used for shaft resistance coefficients. Besides, if the mean-value and COV of the tip resistance coefficients are unknown, values in Table 1 will be used.

When the loading test results are used to estimate the resistance of surrounding piles, attention must be paid to their similarity in ground compositions, SPT-\( N \) value and other properties of ground strength, and pile dimensions like pile length etc., especially in the case of single loading test\(^1\). The reliability of the estimation may not be insured unless the test pile’s condition is similar to surrounding’s condition. To evaluate the similarity, the coefficient of similarity \( J \) in this paper is defined as follows:

\[
J = \gamma_1 \gamma_2
\]

in which, \( J = \text{coefficient of similarity} \). \( \gamma_1 = \text{ratio of the length of estimated pile excluding layers not found in test pile to the total length of estimated pile} \). \( \gamma_2 = \text{ratio of bearing} \)
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## Table 5(a) Estimated values ($\bar{R}_u$) and the estimation errors ($\sigma_R$)

<table>
<thead>
<tr>
<th></th>
<th>P12</th>
<th>P31</th>
<th>P56</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>6.468</td>
<td>6.860</td>
<td>6.860</td>
</tr>
<tr>
<td>from P12 test</td>
<td>$\bar{R}_u$</td>
<td>6.476</td>
<td>3.111</td>
</tr>
<tr>
<td></td>
<td>$\sigma_R$</td>
<td>1.778</td>
<td>0.731</td>
</tr>
<tr>
<td>from P31 test</td>
<td>$\bar{R}_u$</td>
<td>9.550*</td>
<td>6.629*</td>
</tr>
<tr>
<td></td>
<td>$\sigma_R$</td>
<td>2.051</td>
<td>1.483</td>
</tr>
<tr>
<td>from P56 test</td>
<td>$\bar{R}_u$</td>
<td>8.218*</td>
<td>7.432*</td>
</tr>
<tr>
<td></td>
<td>$\sigma_R$</td>
<td>1.507</td>
<td>3.259</td>
</tr>
</tbody>
</table>

unit: MN

☐ : own pile estimation  
* : selected data for CASE-4

## Table 5(b) Estimated values ($\bar{R}_u$) and the estimation errors ($\sigma_R$)

<table>
<thead>
<tr>
<th></th>
<th>P6</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>6.468</td>
<td>6.468</td>
</tr>
<tr>
<td>from P6 test</td>
<td>$\bar{R}_u$</td>
<td>6.194*</td>
</tr>
<tr>
<td></td>
<td>$\sigma_R$</td>
<td>1.651</td>
</tr>
<tr>
<td>from A2 test</td>
<td>$\bar{R}_u$</td>
<td>7.393*</td>
</tr>
<tr>
<td></td>
<td>$\sigma_R$</td>
<td>1.648</td>
</tr>
</tbody>
</table>

unit: MN

☐ : own pile estimation  
* : selected data for CASE-4

The resistance of test pile excluding layers not found in estimated pile to the total bearing resistance of test pile. It could be seen from this definition that $J$ varies in the range of 0 to 1. The bigger the $J$ the more similar between test pile and estimated pile.

The coefficient of similarity $J$ of the five piles are listed in Table 6. It can be seen that the $J$ values related to P12 are very small compared to others. So P12 should be unreliable to estimated other piles or to be estimated by others. Therefore, the estimation related to the P12 are excluded for CASE-4. Based on these results, the first and second order statistics of the error probability for CASE-4 with consideration of estimation similarity are listed in Table 7 where $p = 0.477$ and $\sigma_p = 0.121$, respectively. The probability density function is shown in Fig. 9 with the results of CASE-1 and CASE-3. Finally it is concluded that the estimation accuracy of the bearing resistance by singular loading test taken account of similarity, CASE-4, is approximately the same as that by plural loading tests, CASE-1.
Table 6  The coefficient of similarity for estimation is single loading test (J)

<table>
<thead>
<tr>
<th>A-region</th>
<th>P12</th>
<th>P13</th>
<th>P56</th>
</tr>
</thead>
<tbody>
<tr>
<td>from P12 test</td>
<td>1.00</td>
<td>0.37*</td>
<td>0.71*</td>
</tr>
<tr>
<td>from P13 test</td>
<td>0.27*</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>from P56 test</td>
<td>0.47*</td>
<td>0.92</td>
<td>0.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B-region</th>
<th>P6</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>from P6 test</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>from A2 test</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*: estimation data related to P12

Table 7  The first and second order statistics of error probability \( p \) (CASE-4)

<table>
<thead>
<tr>
<th>Number of data</th>
<th>Mean-value ( \bar{p} )</th>
<th>Standard deviation ( \sigma_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE-4</td>
<td>8</td>
<td>0.477</td>
</tr>
</tbody>
</table>

Both are better than the estimation by current Specifications, CASE-3. Note that the COV of error probabilities for singular loading case is 2.5 times as large as those for plural loading tests case.

5. Performance Factor for the Limit State Design

5.1 Performance factor

When a performance factor in the Limit States Design (reliability-based design level \( I \)) is determined, it is necessary to consider the design level. A method based on the calibration to the current design method is adopted in this paper\(^{23}\). This means that the performance factor for the Limit States Design is determined to be equivalent to the total safety factor in the current design method (Working Stress Design). The design criteria for bearing resistance of piles in the current Working Stress Design is expressed by

\[
\frac{R_u}{n} \leq S_d
\]  \( (19a) \)

in which \( R_u \) = the nominal value (mean-value) of ultimate bearing resistance, \( S_d \) = the nominal value (mean-value) of load at a pile top, and \( n \) = total safety factor.

On the other hand, the design criteria formula in the Limit States Design can be expressed by

\[
\frac{R_u}{F_R} \leq F_S S_d
\]  \( (19b) \)

in which \( F_R \) = resistance factor \( (\geq 1.0) \), \( F_S \) = load factor \( (\geq 1.0) \).

Supposing that \( Z = \ln R_u - \ln S_d \) is the performance function, the allowable safety index \( \beta_a \) is defined as follows\(^{24,25}\).
\[ \beta_a = \frac{\bar{Z}}{\sigma Z} \]  
(20)

Then,
\[ \beta_a = \ln \left( \frac{\bar{R}_u}{\bar{S}_d} \right) \sqrt{\frac{V_R^2}{V_S^2} + V_S^2} \]  
(21)

where \( \bar{R}_u \) and \( V_R \) = mean-value and the COV of ultimate bearing resistances, and \( \bar{S}_d \) and \( V_S \) = mean-value and the COV of load at pile top. Comparing Eq. (19b) with Eq. (21) with introducing the relation \( \sqrt{V_R^2 + V_S^2} = \alpha' (V_R + V_S) \), where \( \alpha' \) is a separation coefficient, both coefficients, \( F_R \) and \( F_S \) are obtained by

\[ F_R = \exp (\alpha' \beta_a V_R) \quad \text{and} \quad F_S = \exp (\alpha' \beta_a V_S) \]  
(22)

If the load factor is independent of the uncertainties of the spatial distribution of the soil properties and the resistance coefficients, these two factors could be conveniently expressed by one performance factor. In other words, \( F_R \) and \( F_S \) can be collectively expressed by one performance factor \( F'_R \) which is the same expression as the total safety factor in the current Japanese Specifications for Substructures.

\[ F'_R = F_R F_S = \exp (\alpha' \beta_a V_R) \exp (\alpha' \beta_a V_S) = \exp (\beta_a \sqrt{V_R^2 + V_S^2}) \]  
(23)

Hence, the factored resistance, \( R_f \), in the Limit states Design and the allowable bearing resistance, \( R_a \), in the Working Stress Design can be expressed by following equations using \( R_u \) and \( F'_R \) or \( n \), with the performance factor, \( F'_R \), being completely equivalent to the total safety factor.

\[ R_f = R_u / F'_R \]  
(24a)

\[ R_a = R_u / n \]  
(24b)

There are two ways to evaluate \( V_R \) in Eq. (23): 1. use of the values in the Japanese Specifications for Substructures, and 2. use of loading test results. The value of \( V_R \) in the former case complies with the results in Table 1 while that in the latter case is expressed by the following equation in which the spatial distribution of SPT-N value and the uncertainty of resistance coefficient are taken into account for the evaluation of the unit shaft resistance \( f_i \):

\[ V_R = \sqrt{V_{R1}^2 + V_{R2}^2} \]  
(25)

in which \( V_{R1} = \text{COV of SPT-N values at the estimation point for the estimation of the spatial distribution of SPT-N values,} \) \( V_{R2} = \text{COV of the resistance coefficients.} \)

The remaining variables at the right term of Eq. (22) are \( \beta_a \) and \( V_a \). The standards in most countries specify \( \beta_a \) for general building and highway bridge structures to be in the range of 2.0-3.52\(^3\). For example the A-58 criterion specifies the value of \( \beta_a \) normally to be 3.0, (2.5 for the case of wind loads and 1.75 for earthquakes\(^2\)). Also, Yamada et al.\(^2\) have proposed that the safety index \( \beta \) due to current design method for the foundation structures in Japan is roughly 3 for an ordinary state (and 1.5 for earthquake loads). These values are adopted in this paper. The COV of the load, \( V_S \), is largely affected by the dead load for an ordinary state, so that a value of 0.1 is used for it because of the small variance of the load. For the case of earthquake loads, there are still many uncertainties in the value of
COV. Here, it is determined by conducting a calibration with the current design method. Based on these concepts, the authors formulate the performance factor taking due account of the variations of the spatial distribution of SPT-N value and the resistance coefficients.

5.2 Discussions

The evaluation method of the performance factor in the Japanese Specifications for Substructures is discussed first. From the COV of the resistance coefficients related to the shaft resistance of the friction pile shown in Table 1, the value of $V_R$ is equal to 0.361. Since the value of $V_{R1}$ in Eq. (25) is equivalent to $V_{N1}$ in Eq. (12), the value of $V_{R1}$ can be expressed as the function of the distance $\lambda$ between a sample point and an estimation point, horizontal correlation parameter $A$, and the COV $V_N$ of the SPT-N value in the layer.

$$ V_{R1} = (0.331 + 0.264\lambda/A) V_N $$  \hspace{1cm} (26)

$V_N$ for the ground may be roughly 0.4 in general. And, in the case of loading test, the position of the test pile nearly corresponds to the position of samples, so that the effect of $\lambda/A$ is neglected. Hence, the value of $V_{R2}$ is obtained as follows based on Eq. (25):

$$ V_{R2} = \sqrt{V_R^2 - V_{R1}^2} = \sqrt{V_R^2 - 0.331^2 V_N^2} = \sqrt{0.361^2 - 0.331^2 \times 0.4^2} = 0.336 $$  \hspace{1cm} (27)

The performance factor based on the current Specifications, $F'_R$, can be expressed by Eq. (28) with substituting Eq. (26) and Eq. (27) into Eq. (23).

$$ F'_R = \exp \left[ \beta_a \sqrt{(0.331 + 0.261\lambda/A)^2 V_N^2 + 0.336^2 + V_S^2} \right] $$  \hspace{1cm} (28)

The relationship between $F'_R$ and $\lambda/A$ obtained from Eq. (28) under the condition that $\beta_a = 3$ and $V_S = 0.1$ (for an ordinary state), and $\beta_a = 1.5$ and $V_S = 0.3$ (for earthquakes), are shown in Fig. 10. Obviously, the performance factor $F'_R$ of both ordinary state and earthquakes for $\lambda/A = 0$ and $V_S = 0.4$ is 3 and 2, respectively, which are the same values as the total safety factor. The performance factor for an ordinary state becomes large as the estimation point gets farther away from the sample point, and this trend is more pronounced as the COV of the SPT-N values of the layer becomes large. As is conditioned in Eq. (12), the COV of estimation, $V_{Nik}$, at the estimation point corresponds to the COV, $V_{N1}$, of that layer when $\lambda/A = 2.5$, because the performance factors become constant in the range of $\lambda/A \geq 2.5$. Compared with the performance factor for an ordinary state, the performance

![Fig. 10 Relation between performance factor ($F'_R$) and normalized distance.](image-url)
factor for earthquakes is less affected by the normalized distance $\lambda/A$ and the $V_{N_1}$ of the 
SPT-$N$ values.

The performance factor taking account of the resistance coefficient based on in-situ 
loading test is discussed next. The value of $V_{R_1}$ at the right hand side of Eq. (25) is 
equivalent to Eq. (26) while the second term $V_{R_2}$ for the case of loading test corresponds to 
the COV of shaft resistance coefficient, $V_{a_1}$, for the case of friction piles. Then Eq. (16) is 
used for the value of $V_{R_2}$. Therefore the value of $V_R$ for the ultimate bearing resistance can 
be expressed by the following equation:

$$V_R = \sqrt{V_{R_1}^2 + V_{R_2}^2} = \sqrt{\left\{ (0.331 + 0.261\lambda/A)^2 V_R^2 + (0.3\overline{a}_{f_1})^2 \right\}} \tag{29}$$

Substituting this equation into Eq. (23), the performance factor $F'_R$ taking account of 
the spatial distribution of the SPT-$N$ value and the loading test data is expressed as

$$F'_R = \exp \left[ \beta_a \sqrt{\left\{ (0.331 + 0.261\lambda/A)^2 V_R^2 + (0.3\overline{a}_{f_1})^2 + V_3^2 \right\}} \right] \tag{30}$$

The performance factor $F'_R$ varies with the mean-value, $\overline{a}_{f_1}$ of the loading test data. 
Here the performance factor $F'_R$ for the case of $\overline{a}_{f_1} = 0.5$ is shown in Fig. 11. The performance 
factor $F'_R$ is approximately 2 at $\lambda/A = 0$ and is about 50% smaller than the value of $F'_R$ 
($= 3$) shown in Fig. 10. This may be attributed to carrying out the loading test to 
increase the reliability of the bearing resistance. Similarly, the performance factor for 
earthquakes can be reduced to 80% of the value of $F'_R$.

6. Application to a Bridge Foundation$^{28}$

6.1 In-situ pile loading tests

The bridge under consideration is a 190m long hollow slab bridge composed of three 
prestressed concrete spans and six reinforced concrete spans. The foundations are supported 
by bored cast-in-place concrete piles of diameter of 1.2m. Fig. 12 shows the soil profile and 
the embedded depth of the foundation piles below the surface of ground at the bridge site.

In order to ascertain how the bearing capacities of friction piles are influenced by such 
differences in pile length and subsurface soil composition, two piles, P2 and P5, having
different lengths were subjected to the in-situ vertical loading tests with the maximum loads $P_{\text{max}}$ of 12MN for P2 and 11MN for P5. Fig. 13 shows the boring logs and the distributions of axial force in the vertical direction. These results apparently indicate that Layer Dc' gives large pile shaft resistance and that the friction piles have very small tip resistance.

Fig. 14 shows the relation between the pile top load $P_0$ and the normalizes settlement $S_0/D$ ($D$: pile diameter) of test piles. The bearing resistances of the piles turned out to be much larger than what had been expected before and the pile settlement under the maximum loading ($P_{\text{max}}$) did not exceed about 2% of the pile diameter in any of the piles. As clarified by Okahara et al.\textsuperscript{5}, the ultimate bearing resistance of a bored pile is generally reached when the pile settlement is equal to about 10% of the pile diameter. If the ultimate bearing resistance is estimated by the Weibull distribution curve equation as proposed by Uto et al.\textsuperscript{29}, it is presumed that the ultimate bearing resistance $R_u$ would be 17MN to 19MN for both the piles.

**Fig. 12** Soil profile and embedded depth of piles.
Fig. 13 Boring logs and axial distributions of test piles.
Fig. 14 Load-normalized settlement curves of test piles.

Fig. 15 shows the relation between the normalized shear resistance $\tau/N_i$ and relative settlement $S$ along the pile shaft for each layer. In this relation, $\tau/N_i$ is a shear resistance divided by an average SPT-$N$ value for that layer, and $S$ is a relative settlement of the pile and the ground in that soil layer. The value of $S$ is calculated using measured strain of pile under linear elastic assumption. $S_i$ and $D_S + D_g$ showed the similar $\tau/N_i$-$S$ curves for both piles P2 and P5. The layers $D_c$ and $D'_c$ were encountered by pile P2 only. The resistance coefficient for $D_c$, which is a layer that spreads continuously and extensively at a level below E1.220m, is remarkably different from that for $D'_c$.

Table 8 shows the resistance coefficients for various layers. Since it is realized from Fig. 15 that $\tau$ seems to gradually approach the peak values, $\tau$ at the time of the maximum loading may be taken as an unit shaft resistance $f_s$. Although two different values of resistance coefficients are obtained for $S_i$ and $D_S + D_g$, since they are characterized by similar $\tau/N_i$-$S$ relation, the value for pile P5 that gave a larger $S$ value was used as design values.

Fig. 15 Normalized shear resistance-relative settlement curves for each layer.
almost solely by the magnitude of its ultimate bearing resistance and is irrelevant to the uncertainty of the soil properties. On the other hand, the evaluation of $R_f$ is dependent on the uncertainty of the $SPT-N$ values of each foundation. For example, in the case of pile P3, $R_f$ is underevaluated compared to $R_a$. This is because the $SPT-N$ value of $D_c$ and $D_s + D_c$ below it are estimated on the basis of the sample values at the positions of piles P2 and P5 and consequently these $SPT-N$ values are liable to large estimation errors which makes the performance factors of these layers higher. In the same way, pile P8 is assessed to have a factored bearing resistance smaller than the allowable bearing resistance because no soil investigation was conducted at its location. The same comment can also be made about the bearing resistance for earthquakes.

Fig. 18 shows the ratios of the load $S_d$ to the bearing resistance $R_f$ or to $R_a$, of each foundation. $S_d/R_f$ is smaller than $S_d/R_a$ for all foundations, either in an ordinary state or
Table 8  Resistance coefficients for each layer

<table>
<thead>
<tr>
<th>Layer</th>
<th>Pile</th>
<th>Mean-value</th>
<th>Unit shaft resistance</th>
<th>Resistance coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\bar{N}_i$</td>
<td>$f_i$ (kPa)</td>
<td>$\alpha_n$</td>
</tr>
<tr>
<td>Si</td>
<td>P2</td>
<td>13.0</td>
<td>38.9</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>17.9</td>
<td>72.4</td>
<td>0.413</td>
</tr>
<tr>
<td>Ds+Dg</td>
<td>P2</td>
<td>41.0</td>
<td>160</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>27.8</td>
<td>127</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>45.5</td>
<td>227</td>
<td>0.511</td>
</tr>
<tr>
<td>Dc</td>
<td>P2</td>
<td>11.0</td>
<td>284</td>
<td>2.63</td>
</tr>
<tr>
<td>Dc'</td>
<td>P2</td>
<td>13.8</td>
<td>77.5</td>
<td>0.573</td>
</tr>
</tbody>
</table>

Table 9  The first and second order statistics of N-values for each layer

<table>
<thead>
<tr>
<th>Layer</th>
<th>Sample size</th>
<th>Mean-value $\bar{N}_i$</th>
<th>COV $V_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>85</td>
<td>18.0</td>
<td>0.364</td>
</tr>
<tr>
<td>Ds+Dg</td>
<td>84</td>
<td>41.8</td>
<td>0.452</td>
</tr>
<tr>
<td>Dc</td>
<td>39</td>
<td>11.2</td>
<td>0.413</td>
</tr>
</tbody>
</table>

6.2 Estimation of spatial distribution of SPT-N values

Table 9 shows the first and second order statistics of the SPT-N values in each layer of the ground. The COV of each layer is approximately 36 to 45%, and this is within a normal range for a SPT-N value. Fig. 16 shows the estimated spatial distribution of SPT-N values of $S_i$ and $D_c$ as obtained by the statistical analysis where the correlation parameter $A = 10m$. In this connection, estimation errors were obtained by multiplying the COV of estimation $V_{N_i}$ by the mean value of estimation $\bar{N}_i$. This figure shows that there are no soil investigation between boring points c and e in the case of the $D_c$ layer. As a result, the estimation errors for any points between them become inevitably large.

6.3 Evaluation of bearing resistances of piles

The ultimate bearing resistance $R_u$, the factored bearing resistance $R_f$ and the allowable bearing resistance $R_a$ for each foundation as obtained by Eq. (24a) and (24b) are shown in Fig. 17. The total safety factors used in calculating allowable bearing resistances are taken as 3 for Ordinary state and 2 for earthquakes as suggested by Specifications for Substructures. Since the allowable bearing resistance for Ordinary state is equal to the ultimate bearing resistance divided by 3 (i.e. $R_u/3$), the bearing resistance of each foundation is determined
earthquakes. Thus, it is believed that the use of the proposed performance factors enables the safety of bearing resistance to be assessed in a more rational manner and makes it possible to achieve a more economical design.

7. Concluding Remarks

The semi-empirical bearing resistance expression for bored friction piles was focused on, and a performance factor for the Limit State Design of these piles was proposed taken into account the uncertainties of $SPT-N$ value and the resistance coefficient.

The main conclusions drawn from this study are summarized as follows:

1. The Spatial distribution of $SPT-N$ value was well estimated using the Kriging technique and the simplified expression for this estimation method was proposed in order to formulate the performance factor, explicitly.

2. The difference of the quantitative evaluation of the resistance coefficient with and without in-situ loading test was clarified. The $COV$ of the resistance coefficient, $V_{ai}$, was estimated using its mean-value $\bar{a}_i$ when the in-situ loading test was conducted.

3. The performance factor taking into account both the uncertainties on $SPT-N$ value and the resistance coefficient was expressed by proposed normalized parameter $\lambda/A$ and the $COV$ of $SPT-N$ value, $V_N$.

4. It was demonstrated that the proposed performance factor provides more rational and efficient basis for bridge foundation design.

Finally, the proposed performance factor could evaluate the reliability of the bearing capacity of bored friction piles quantitatively so that this factor could suitably applied in Limit State Design procedures.

References

1) Japan Road Association, Specifications for Substructures, Specifications for Highway Bridges (Part. 4), 1990.


