Bearing Capacity of Shallow Foundation with Stepped Footing on Slopes

by

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Abstract

Bearing capacity formulas for shallow foundations with stepped footing on slopes are proposed from theoretical and experimental studies. The bearing capacity formulas are derived from the velocity field method based on the upper bound theorem of plasticity theory, and are capable of considering the effects of stepping shapes on foundation bottom face, ground self-weight and inclination of load. Loading tests of bearing capacity are carried out to verify the proposed formulas. The tests consist of model loading tests in laboratory on soil cement, Toyoura standard sand and undisturbed Shirasu. A series of large-scale in-situ loading tests is also conducted on natural in-situ Shirasu grounds as well as to prove the usefulness of the centrifugal loading tests. The test results show the validity of the proposed formulas.

Keywords: Bearing capacity, Centrifugal model test, In-situ loading test, Limit analysis, Shallow foundation, Slope, Stepped footing

1. Introduction

In constructing important civil engineering structures such as expressways in Japan, where mountainous areas account for about 75% of the land, cases where the construction must be done on steep are sharply on the rise. Accordingly, the importance of researches relating to evaluating bearing capacity of structural foundations built on slopes has grown.

This paper readily adaptable bearing capacity formulas relating to shallow foundations on slopes. On these matters several analytical studies have been performed in the past[123]. However, all previous studies have dealt with flat shallow foundations constructed near slope shoulders.

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In practice, "stepped footing" is frequently used to reduce the amount of soil excavation from the viewpoint of slope ground stabilization and environmental maintenance. However, there are very few studies on such stepped footing where the foundation bottom and the bearing ground have more than one contact face with the ground. Furthermore, most of the all past studies place constraints on the influence of the ground self weight and the influence of the load inclination, and have limitations in terms of adaptability.

In relation to the bearing capacity of shallow foundations on slopes, this paper proposes flexible bearing capacity formulas that take into account ① the influence of the shape of the stepped footing; ② the influence of the ground self weight; and ③ the influence of load inclination. The velocity field method is used in this report. And, the formulas are then verified experimentally.

2. Proposed bearing capacity formulas taking step shapes into account

2.1 Stepped footing on a slope and basic failure mechanisms

2.1.1 Stepped footing shape and specificity of its bearing capacity mechanism

Stepped footing on a slope is generally adopted in the case where the bearing stratum is shallow; it often takes the shapes shown in Fig. 1. As shown, the number of steps is mostly up to two, and the height and width of stepping per step is generally about 3 \( (m) \) or less. Such footings as (a) and (b) above are generically called "stepped footing".

To indicate the shapes of these stepped footings, this paper uses such indications as shown in Fig. 2.

Describing the stepped footing shapes shown in Fig. 2 requires four items: total footing width, lower footing width, stepping gradient, and stepping height. The stepping gradient often takes nearly a constant value of 30% to 50% (the ratio of the vertical height to the horizontal distance is 1:0.3 to 1:0.5) depending on kind of natural ground condition. Therefore, the following three parameters are thought important for practical use and are used in the discussions that follow:

Fig. 1 Stepped footing on slope

Fig. 2 Delineation of stepped footing shape
Fig. 3 Isolation of stepped footing from analysis

- $m =$ stepping height/total footing width (B)
- $n =$ stepping width/total footing width (B)
- $\rho =$ margin width from top of slope/total footing width (B)

Practical designs ordinarily use about 3 ($m$) or less for the stepping height per step and the stepping width, and 4 to 10 ($m$) for the foundation footing width B in shallow foundations. Therefore, m and n per step for varification purposes may be assumed to be approximately 0.3 to 0.7. If the foundation bottom is flat, $m=0$ and $n=0$ may be used.

With respect to the margin width from the top of a slope ($\rho$ in Fig. 2), there are studies that explain it almost purely theoretically. Therefore, we will proceed with our discussion assuming the case where there is no isolation from the top of the slope.

Stepped footing has a number of contact faces with the ground. We will discuss the basic pattern for failure mechanisms using as a model the stepped footing shown in Fig. 3. If a footing has steps ($m \neq 0$, $n \neq 0$), the failure mechanism is anticipated to vary according to the degrees of the steps. We will separate this problem into the flat section (line O-A in Fig. 3) and the step section (lines A-B-C).

Fig. 3 (b) notes the flat section at the lower edge. The failure mechanism in this case is the same as that proposed by Kusakabe\(^a\). On the other hand, the stepped footing shown in Fig. 3 (c) poses a problem fairly different from that in Fig. 3 (b) in terms of dynamics, and a basic failure mechanism has not been attempted to date.

It can be anticipated that an increase in the upper load $Q_1$ may generate active soil pressure in the ground, which can further increase the deformation of the foundation to the valley side. This behavior is believed to have a close relation with the bearing capacity characteristics of the stepped footing.

2.1.2 Basic failure mechanism and admissible velocity field of L-shaped footing

To simplify the problem of stability in stepped footing, the L-shaped footing shown in Fig. 4 is considered. Also, a case is considered where an upright slope face, whose foundation width B has distributed load "q" at its horizontal boundary, is supported with a side wall having of length mB. The regions $\tilde{I}$ and $\tilde{I}$ are rigidly integrated footing sections, and hence their displacement velocity components are equivalent, and must be:
Fig. 4 Failure mechanism in L-shaped footing and permissible velocity field

\[ V_{x_1} = V_{x_B} \]  \hspace{1cm} (1)

\[ V_{y_1} = V_{y_B} \]  \hspace{1cm} (2)

However, the velocity components perpendicular to the face BC must be equal for them to be an admissible velocity field. Therefore:

\[ V_{y_1} = V_{y_B} : \text{on face BC} \]  \hspace{1cm} (3)

On the one hand, the relation among the velocity components of \( V_{x_1} \), \( V_{x_B} \) and \( V_{x_B} \) can incorporate different admissible velocity fields based on a hypothetical condition for the behavior of the foundation toward the valley side. This problem is discussed specifically later on.

(1) Where horizontal movement is generated in foundation

Fig. 5 shows the failure mechanism and admissible velocity field when the foundation subsides integrally with the ground, accompanied with a horizontal movement toward the valley side. The following equation should be adopted in addition to the Eqs. (1) through (3):

\[ V_{x_B} = V_{x_B} : \text{on face AB} \]  \hspace{1cm} (4)

Therefore, the admissible velocity field will be:

\[ V_{x_1} = V_{x_B} = V_{x_B} \]  \hspace{1cm} (5)

\[ V_{y_1} = V_{y_B} = V_{y_B} \]  \hspace{1cm} (6)

Thus, regions I, II, and III have equivalent velocity components.

If the dilatancy angle \( \nu \) is assumed to be equivalent to the internal friction angle \( \phi \) according to the hypothesis in the associated flow rule, the following failure mechanism is possible:

\[ V_{x_1} = V_{x_B} = V_{x_B} = -V_\delta \sin \delta = -V_1 \cos (\phi - \phi) \]  \hspace{1cm} (7)
where $V_A$ is the hypothetical velocity of foundation displacement toward the lower direction of the valley side, $V_0$ is the displacement velocity component in the vertical direction in region I (this is the displacement velocity due to external work), and $V_1$ is a displacement velocity on the slip line, having an angle of $\phi$ between the slip-line and the internal friction angle.

These can be shown as in Fig. 5, in which regions I, II, and III move toward the lower left-hand side with an identical velocity component ($V_A = V_1$). The failure mechanism that satisfies this admissible velocity field is hereinafter referred to as "mechanism A".

On the other hand, if the stepping height is high enough, we may assume that the slip-line created from point C crosses AB, the failure mechanism at this time being shown in Fig. 6. To constitute an admissible velocity field, the foundation on face BD and the soil mass $\Delta BCD$ can't be superposed, but may be in constant contact or may be separated. If $\cos \delta = \frac{V_0}{V_A}$, $V_0 = V_1 \sin (\phi - \phi)$, the following signs of inequality must have been established between the foundation and the horizontal velocity components of soil mass $\Delta BCD$, in addition to the Eqs. (1) through (3):

$$V_A \sin \delta = V_0 \tan \delta \geq V_1 \cos (\phi - \phi) = V_0 \frac{1}{\tan (\phi - \phi)}$$

Or,

$$\tan \delta \geq \frac{1}{\tan (\phi - \phi)}$$

At this time, the angle $(\phi - \phi)$ is at its maximum:

$$(\phi - \phi)_{\text{max}} = \tan^{-1} \frac{m}{1 - n}$$

and when $(\phi - \phi)$ takes the maximum value, a slip that passes through the lower edge of the stepped footing will occur. When $(\phi - \phi) < (\phi - \phi)_{\text{max}}$, a clearance having the width expressed in the following equation will be created between the foundation and the ground along face BC:

**Fig. 5** Mechanism A, where horizontal movement is permitted

**Fig. 6** Mechanism A', where horizontal movement is permitted
\[ \Delta V_{bd} = V_0 \tan \delta - V_1 \cos (\phi - \phi) = V_0 \left[ \tan \delta - \frac{1}{\tan (\phi - \phi)} \right] \] (10)

Generally, the foundation bottom is coarse enough, and the soil mass existing above the slip line can be assumed to behave in the same way as the footing. Therefore, the discontinuity of the displacement velocity under this condition can be disregarded.

In this case, Eq. (10) is \( \Delta V_{bd} = 0 \).

Therefore, the following equation can be derived from Eqs. (9) and (10) as the conditional equation when the foundation and the ground directly below the foundation behave in an identical fashion:

\[ \tan \delta \cdot \tan (\phi - \phi) = 1 \] (11)

The failure mechanism that satisfies this admissible velocity field is referred to as mechanism A'.

If the foundation and the ground directly below it behave integrally, the above considerations reveal that external work against the horizontal direction would not occur under the action of the vertical load, even if the foundation accompanies a movement toward the valley side. Therefore, it is enough to consider the velocity component \( V_0 \) in the load direction as perpendicular.

(2) Where no horizontal movement is generated in the foundation

We may also consider a failure mechanism (Fig. 7) that satisfies the conditional Eqs. (5) and (6) for the case where the foundation behaves integrally with the ground.

Fig. 7  Mechanism B, where horizontal movement is fixed
\[ V_{x1} = V_{xg} = V_{xb} = 0 \]  \hspace{1cm} (12)
\[ V_{y1} = V_{yg} = V_{yb} = V_0 \]  \hspace{1cm} (13)

In this case, all of the regions \( I \), \( II \), and \( III \) have only a vertical displacement velocity component. \( \triangle ABC \) moves downward as a rigid body as shown in Fig. 7, and its horizontal movement is fixed. This type of failure mechanism requires other moving regions below region \( III \). This failure mechanism, which satisfies the admissible velocity field, is hereinafter referred to as mechanism B. However, in practical construction design, with this mechanism it would be difficult to satisfy Eq. (12), and to this mechanism may be thought to constitute a very special case.

2. 2 Bearing capacity formulas for stepped footing according to the velocity field method

As a failure mechanism for the case where stepped footing causes a failure over the entire foundation, it would make sense to use Fig. 3 (b) (discussed in 2.1) together with the basic mechanisms in Figs. 5, 6, and 7.

In the following discussion, an admissible velocity field is set for these specific failure mechanisms, and bearing capacity formulas are proposed. The discussion in the text will deal with mechaism A and mechanism A' from the standpoint of actual design work. However, bearing capacity formulas may be obtained for mechanism B in the same manner.

2. 2. 1 Failure mechanism A

This failure mechanism is obtained through use of Fig. 3 (b) and Fig. 5, forming the admissible velocity field as shown in Fig. 8. The slip-lines in the active section caused from the foundation edge "c" will reach the passive region via the transition region similarly to the case where the foundation is flat, while crossing the slip-lines from the specific points "o", "a", and "b" at other stresses.

In this case, points "a" and "b" are the specific points of the stress, but point "b" has the ground surrounded by the footing, and thus a lateral displacement of only the ground is not permitted. Because the ratio of the vertical stress to the horizontal stress in the ground is thought to change very little even if the lateral pressure in the horizontal direction changes due to an increased vertical load, the condition becomes similar to the stress path for one-dimensional compaction where shear failure would not occur. Therefore, slip-lines having point "b" as the starting point are considered not to occur. On the one hand, if the hand, if the slip-line in "ce" can be hypothesized to be a straight line, "ab" will also be a straight line. However, since these two have an intersection angle of \( (\pi/2 - \phi) \), the "ad" will cross perpendicularly with a displacement velocity \( V_1 \). Therefore, the velocity would not develop a discontinuity, and internal dissipation will disappear. Thus, the slip-line in "ad" is also thought not to occur. This line of reasoning has been verified experimentally, as described later.

The steady-state condition for the admissible velocity field in mechanism A is as follows:

1. Steady-state condition for "bc" and "oa".
\[ V_0 = V_1 \sin(\phi - \phi) \]

2. Steady-state condition on both sides of the transition region "oef".
Fig. 8 Failure mechanism and admissible velocity field (mechanism A)

\[ V_3 = V_1 \exp(\theta \cdot \tan \phi) \]

Since "oef" is logarithmic spiral, Substituting \( V_0 \) from the above results,

\[ V_3 = \frac{V_0 \cdot \exp(\theta \cdot \tan \phi)}{\sin(\phi - \phi)} \]

where \( \theta \) is set against a certain "h", and the relation between \( \theta \) and "h" is shown by the following equation:

\[
h = [1 - m \cdot \tan(\pi/2 - \phi)] \cdot \sin \phi
\frac{\sin(\pi/2 - \phi + \phi - \theta) - \cos(\pi/2 - \phi + \phi - \theta) \cdot \tan(\phi - \theta)}{\sin(\pi/2 - \phi) \cdot [1 - \cos\beta \cdot \tan(\phi - \theta)]}
\cdot \exp(\theta \cdot \tan \phi)
\]

(14)

The admissible velocity field is thus determined. Then, the internal dissipation energy along each discontinuity line in the admissible velocity fields can be obtained by substituting the admissible velocity into the following equation.

\[ E_{ce} = c \cdot \overline{ce} \cdot V_1 \cdot \cos \phi \]
\[ E_{oef} = c \cdot \overline{oef} \cdot V_1 \cdot \cot \phi \cdot [\exp(2\theta \cdot \tan \phi) - 1] \]
\[ E_{fg} = c \cdot \overline{fg} \cdot V_3 \cdot \cos \phi \]
Therefore, the total internal dissipation energy is presented as the sum of each internal dissipation energy, which is given by:

\[ E_{\text{total}} = E_{ce} + E_{oef} + E_{fg} \]

where the equation \( E_{\text{total}} \) can be tied with \( c \cdot V_0 \cdot B \). If the remaining term is defined as \( N_c \), \( E_{\text{total}} \) can be expressed as:

\[ E_{\text{total}} = c \cdot V_0 \cdot B \cdot N_c \]

On the one hand, the external work due to the load and ground self weight is calculated as follows:

External work due to load "q"

\[ q \cdot V_0 \cdot B \]

External work due to ground self weight of soil mass

\[ W_{abcd} = -\frac{r}{2} \left[ (bc + ad') \cdot m \cdot B \right] V_1 \cdot \sin(\phi - \phi) \]

\[ W_{o\theta a} = -\frac{r}{2} \left[ d'e \cdot od' \cdot \sin\phi \cdot V_1 \cdot \sin(\phi - \phi) \right] \]

\[ W_{oef} = \frac{r}{2} \int_0^\theta \left[ oe \cdot \exp(\theta \tan \phi) \right]^2 V_1 \cdot \exp(\theta \tan \phi) \cdot \sin(\phi - \phi - \theta) \, d\theta \]

\[ W_{ofg} = \frac{r}{2} \left[ of \cdot og \cdot \sin(\pi/2 - \phi + \phi - \beta - \theta) \right] \cdot V_3 \cdot \sin(\phi - \phi - \theta) \]

Therefore, the total external work is expressed as follows:

\[ W_{\text{total}} = q \cdot V_0 \cdot B + W_{abcd} + W_{o\theta a} + W_{oef} + W_{ofg} \]

\[ = q \cdot V_0 \cdot B + \Sigma W \]

where the equation of \( \Sigma W \) can be tied with \( -\gamma \cdot V_0 \cdot B^3/2 \). If the remaining term is defined as \( N_r \), \( W_{\text{total}} \) can be expressed as:

\[ W_{\text{total}} = q \cdot V_0 \cdot B - \gamma \cdot V_0 \cdot B^3/2 \cdot N_r \]

Since the total external work \( W_{\text{total}} \) and the total internal dissipation energy \( E_{\text{total}} \) are equivalent, the bearing capacity evaluation for the mechanism A is obtained as follows:

\[ W_{\text{total}} = E_{\text{total}} \]

That is:

\[ q \cdot V_0 \cdot B - \gamma \cdot V_0 \cdot B^3/2 \cdot N_r = c \cdot V_0 \cdot B \cdot N_c \]

When "q" is sought from this equation:

\[ q = c \cdot N_c + \frac{\gamma \cdot B}{2} \cdot N_r \]

where the bearing capacity coefficients \( N_c \) and \( N_r \) are given by the Eqs. (16) and (17).

\[ N_c = \frac{\cos \phi}{\sin (\phi - \phi)} \]
\[
\begin{align*}
\cdot \left( \frac{m}{\sin \phi} + \left[ 1 - m \cdot \tan (\pi/2 - \phi) \right] \cdot \sin (\pi/2 + \phi - \psi) \right) \\
+ \frac{\cos \phi \cdot \exp (\theta \cdot \tan \phi)}{\sin (\phi - \phi) \cdot \cos \phi \cdot (\pi/2 - \phi)} \cdot \{ h \cdot \cot \beta \} \\
+ \left[ 1 - m \cdot \tan (\pi/2 - \phi) \right] \cdot \sin \phi \cdot \cos (\pi/2 - \phi \cdot \phi - \theta) \\
- \frac{\exp (\theta \cdot \tan \phi)}{\sin (\pi/2 - \phi) \cdot \sin (\phi - \phi)} \cdot [1 - m \cdot \tan (\pi/2 - \phi)] \cdot \sin \phi \cdot \cot \phi \\
\cdot \left[ \exp (2 \theta \cdot \tan \phi) - 1 \right]
\end{align*}
\]

\[N_r = - \left\{ m \cdot \left[ 2 - 2n - m (\cot \phi + \cot \beta) \right] \right\} + \left[ 1 - m \cdot \tan (\pi/2 - \phi) \right] ^2 \cdot \sin (\pi/2 + \phi - \phi) \cdot \sin \phi \\
+ h \cdot \left[ 1 - m \cdot \tan (\pi/2 - \phi) \right] \cdot \sin \phi \cdot \sin (\pi/2 - \phi + \phi - \beta - \theta) \\
\cdot \sin (\phi - \theta - \phi) \cdot \exp (2 \theta \cdot \tan \phi) \\
+ \left[ 1 - m \cdot \tan (\pi/2 - \phi) \right] ^2 \cdot \sin ^2 \phi \cdot \sin (\phi - \phi - \phi) \cdot \sin (\phi - \phi) \cdot \sin (\psi - \phi) \cdot \sin ^2 (\pi/2 - \phi) \cdot \sin (\phi - \phi) \cdot I\right] \]

where I = \[ \int_0^\infty \exp (3 \theta \cdot \tan \phi) \cdot \sin (\phi - \phi - \theta) \] d\theta can be developed into the following equation:

\[ I = \{ \exp (3 \theta \cdot \tan \phi) \left[ 3 \tan \phi \cdot \sin (\phi - \phi - \theta) + \cos (\phi - \phi - \theta) \right] \] \\
- \left[ 3 \tan \phi \cdot \sin (\phi - \phi) + \cos (\phi - \phi) \right] / \left( (3 \tan \phi) ^2 + 1 \right) \]

To derive the bearing capacity, since the active wedge angle \( \phi \) and the angle \( \theta \) indicating the transition region are unknown, the above calculations are repeated, with the resultant smallest value taken as the bearing capacity. The wedge angle \( \phi \) may be calculated between \( \phi = \phi \) of Terzaghi and \( \phi = \pi/4 + \phi/2 \) of Prandtl. Because the proposed formulas take into consideration the ground self weight and the stepping shape, the wedge angle \( \phi \) that provides the minimum bearing capacity due to changes in the ground self weight and the stepping shape varies.

2. 2. 2 Failure mechanism A’

This failure mechanism occurs when the step height is high. It is obtained using Fig. 3 (b) and Fig. 6; results are shown in Fig. 9. This mechanism A’ can be formed when \( 1 - n - m \cdot \cot \phi > 0 \) according to the geometrical condition, and its bearing capacity value derives from mechanism A in the case where \( 1 - n - m \cdot \cot \phi > 0 \) and at \( 1 - n - m \cdot \cot \phi = 0 \). When Eq. (10) can be formed, clearances can develop along the "bd" and "ad" planes. However, since the foundation bottom is generally coarse enough, \( \Delta bcd \) and \( \Delta oae \) and the foundation behave integrally, and the clearance can develop only on plane "ad".

In this case, \( \Delta bcd \), and \( \Delta oae \) contact the foundation at "be" and "oa" respectively, and the displacement velocity in the perpendicular direction is the same \( V_9 \) as that for the respective foundation. Its horizontal direction components will also have the same value.
because $\Delta bcd$, and $\Delta oae$ respectively move intergrally with the foundation. In other words, the displacement velocities for $\Delta bcd$, and $\Delta oae$ have the same magnitude and direction, and the relation $V_1 = V_2$ is formed. Further, since the displacement velocity forms an angle $\phi$ against the discontinuity plane (slip-line), $\phi_1$ and $\phi_2$ (Fig. 9) also become equal.

For the above reasons, our discussion will continue taking the conditions $V_1 = V_2$, and $\phi_1 = \phi_2$ as prerequisites.

The condition of the admissible velocity field for the failure mechanism $A'$ shown in Fig. 9 to be formed is as follows:

1. Steady-state condition at "bc"

$$V_0 = V_1 \cdot \sin (\psi_1 - \phi)$$

where, if $V_1 = V_2$, $\psi_1 = \psi_2$,

$$V_0 = V_2 \cdot \sin (\psi_2 - \phi)$$

2. Steady-state condition at "oa"

$$V_0 = V_2 \cdot \sin (\psi_2 - \phi)$$

3. Steady-state condition on both sides of the transition region "oef". Since "oef" is a logarithmic spiral,

$$V_3 = V_2 \cdot \exp (\theta \cdot \tan \phi)$$

substituting $V_0$ from the previous result,
\[ V_3 = \frac{V_0 \cdot \exp(\theta \cdot \tan \phi)}{\sin(\phi - \phi)} \]

where \( \theta \) is set against a certain "h", and the relation between \( \theta \) and "h" is shown by the following equation:

\[ h = n \cdot \sin \phi_2 \]

\[ \cdot \frac{[\sin(\pi/2 - \phi + \phi_2 - \theta) - \cos(\pi/2 - \phi + \phi_2 - \theta) \cdot \tan(\phi_2 - \theta)]}{\sin(\pi/2 - \phi) \cdot [1 - \cot \beta \cdot \tan(\phi_2 - \theta)]} \cdot \exp(\theta \cdot \tan \phi) \]

Thus the admissible velocity field has been determined. Then, the internal dissipation energy along each line of discontinuity in the admissible velocity fields can be obtained by substituting the admissible velocity into the following equation:

\[ E_{cd} = c \cdot cd \cdot V_1 \cdot \cos \phi \]

\[ E_{ae} = c \cdot ae \cdot V_2 \cdot \cos \phi \]

\[ E_{oef} = c \cdot oe \cdot V_2 \cdot \cot \phi \cdot [\exp(2 \theta \cdot \tan \phi) - 1] \]

\[ E_{fg} = c \cdot fg \cdot V_3 \cdot \cos \phi \]

Therefore, the total internal dissipation energy is presented as:

\[ E_{total} = E_{cd} + E_{ae} + E_{oef} + E_{fg} \]

where the equation of \( E_{total} \) can be tied with \( c \cdot V_0 \cdot B \cdot N_c \), and if the remaining term is defined as \( N_c \), \( E_{total} \) can be expressed as:

\[ E_{total} = c \cdot V_0 \cdot B \cdot N_c \]

On the one hand, the external work due to the load and ground self weight is calculated as follows:

External work due to load "q"

\[ q \cdot V_0 \cdot B \]

External work due to the ground self weight of the soil mass

\[ W_{bcd} = \frac{r}{2} \cdot bc \cdot cd \cdot \sin \phi_1 \cdot V_1 \cdot \sin(\phi_1 - \phi) \]

\[ W_{oae} = \frac{r}{2} \cdot oe \cdot ae \cdot \sin \phi_2 \cdot V_2 \cdot \sin(\phi_2 - \phi) \]

\[ W_{oef} = \frac{r}{2} \cdot \int_0^\theta \left[ oe \cdot \exp(\theta \cdot \tan \phi) \right]^2 \cdot V_2 \]

\[ \cdot \exp(\theta \cdot \tan \phi) \cdot \sin(\phi_2 - \theta - \phi) \mathrm{d}\theta \]

\[ W_{ofg} = \frac{r}{2} \cdot \left[ of \cdot og \cdot \sin(\pi/2 - \phi + \phi_2 - \beta - \theta) \right] \]

\[ \cdot V_3 \cdot \sin(\phi_2 - \theta - \phi) \]

Therefore, the total external work is expressed as follows:
\[ W_{\text{total}} = q \cdot V_0 \cdot B + W_{\text{bed}} + W_{\text{ase}} + W_{\text{ol}} + W_{\text{of}} \]
\[ = q \cdot V_0 \cdot B + \sum W \]

where the equation of \( \sum W \) can be tied with \(- \gamma \cdot V_0 \cdot B^2 / 2\), and if the remaining term is defined as \( N_r \), \( W_{\text{total}} \) can be expressed as:
\[ W_{\text{total}} = q \cdot V_0 \cdot B - \gamma \cdot V_0 \cdot B^2 / 2 \cdot N_r \]

Since the total external work \( W_{\text{total}} \) and the total internal dissipation energy \( E_{\text{total}} \) are equal, the bearing capacity evaluation equation for mechanism A' is obtained as follows:
\[ W_{\text{total}} = E_{\text{total}} \]

That is:
\[ q \cdot V_0 \cdot B - \gamma \cdot V_0 \cdot B^2 / 2 \cdot N_r = c \cdot V_0 \cdot B \cdot N_c \]

When "q" is sought from this equation:
\[ q = c \cdot N_c + \frac{\gamma \cdot B}{2} \cdot N_r \]

(19)

Where the bearing capacity coefficients \( N_c \) and \( N_r \) are given by Eqs. (20) and (21).

\[ N_c = \frac{(1-n-m \cdot \cos \beta_i) \cdot \sin(\pi - \beta_i) \cdot \cos \phi}{\sin(\beta_i - \phi) \cdot \sin(\phi - \phi)} + \frac{n \cdot \sin(\pi / 2 + \phi - \phi_2) \cdot \cos \phi}{\sin(\pi / 2 - \phi) \cdot \sin(\phi_2 - \phi)} + \frac{\cos \phi \cdot \exp(\theta \cdot \tan \phi)}{\sin(\phi_2 - \phi) \cdot \cos(\phi_2 - \phi)} \cdot \left[ h \cdot \cot \beta + n \cdot \sin \phi_2 \cdot \cos(\pi / 2 - \phi + \phi_2 - \theta) \cdot \exp(\theta \cdot \tan \phi) \right] \]
\[ + \frac{n \cdot \sin \phi_2 \cdot \cot \phi \cdot [\exp(2 \theta \cdot \tan \phi) - 1]}{\sin(\pi / 2 - \phi) \cdot \sin(\phi_2 - \phi)} \]  \hspace{1cm} (20)

\[ N_r = - \left[ \frac{(1-n-m \cdot \cot \beta_i)^2 \cdot \sin(\pi - \beta_i) \cdot \sin \phi_2}{\sin(\beta_i - \phi_2)} + \frac{n^2 \cdot \sin(\pi / 2 + \phi - \phi_2) \cdot \sin \phi_2}{\sin(\pi / 2 - \phi)} + \frac{n \cdot h \cdot \sin \phi_2 \cdot \sin(\pi / 2 + \phi_2 - \beta - \theta) \cdot \sin(\pi / 2 - \phi)}{\sin(\pi / 2 - \phi) \cdot \sin(\pi / 2 - \phi)} \cdot \exp(2 \theta \cdot \tan \phi) + \frac{n^2 \cdot \sin^2 \phi_2}{\sin^2(\pi / 2 - \phi) \cdot \sin(\phi_2 - \phi)} \cdot I \right] \]
\[ \text{where,} \]
\[ I = \frac{\exp(3 \theta \cdot \tan \phi) \cdot [3 \tan \phi \cdot \sin(\phi_2 - \phi - \theta) + \cos(\phi_2 - \phi - \theta)]}{(3 \tan \phi)^2 + 1} \]
\[ - \frac{3 \tan \phi \cdot \sin(\phi - \phi) + \cos(\phi - \phi)}{(3 \tan \phi)^2 + 1} \]

2.2.3 Result of calculating the bearing capacity

This section considers the influence of the foundation shape on the bearing capacity by proposed formulas:

(1) Relation between the changes in active wedge angles and the bearing capacity
Fig. 10 is the result of a study on the relation between the active wedge angle $\phi$ at the failure mechanisms A and A' and the bearing capacity "q". It uses stepping height ratios of $m=0, 0.3, 1.0$, and internal friction angles of $\phi=20^\circ, 30^\circ, 40^\circ$, and takes the ground self weight into account. To compare the ground self weight with the analytical equation, $\gamma=0$ and $\gamma=2t/\text{f/m}^3$ are used. The latter is used most often in calculating the bearing capacity as a function of soil character and foundation. This clarifies the following: the wedge angle $\phi_{q(\text{min})}$ that gives the smallest bearing capacity at the same stepping height ratio "m" and the internal friction angle $\phi$ varies depending on the ground self weight; the wedge angle $\phi_{q(\text{min})}$ decreases when ground self weight is taken into consideration.

On the one hand, at the same ground self weight $\gamma$ and internal friction angle $\phi$, the wedge angle $\phi_{q(\text{min})}$ decreases as the stepping height ratio "m" increases. However, the variation in the bearing capacity in the vicinity of $\phi_{q(\text{min})}$ is not noticeably large. As a result of a decrease in the wedge angle $\phi$, the displacement velocity of the foundation ground requires a larger horizontal component to obtain the perpendicular bearing capacity (or the displacement velocity $V_0$) of the same degree. This means that the foundation tends to create a horizontal movement more easily with the increase in the stepping height at the stepped footing.

Fig. 11 shows the relation between the wedge angle $\phi_{q(\text{min})}$ to giving the smallest bearing capacity and the stepping height ratio "m". As described earlier, the $\phi_{q(\text{min})}$ gets

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**Fig. 10**  Variation in bearing capacity due to active wedge angle (mechanisms A, and A')
smaller with the presence of the ground self weight, but as the but as the slope angle $\beta$ decreases, $\psi_{q(\text{min})}$ gets still smaller because of the contribution of the ground self weight term to the bearing capacity increases.

According to the bearing capacity analysis on a flat dry sand ground, Kimura et al. have shown that the active wedge angle that gives the smallest bearing capacity in terms of the analysis is close to the Meyerhof's proposal ($\psi_{q(\text{min})} = 1.2\phi$).

In stepped footing, $\psi_{q(\text{min})}$ gets smaller as the stepping height ratio "m" increases, even if the ground self weight is disregarded $\gamma = 0$. The only exception is that $\psi_{q(\text{min})}$ becomes an equivalent value irrespective of the slope angle $\beta$ when there is no ground self weight ($\gamma = 0$) and the bottom face is flat ($m = 0, n = 0$), that is Prandtl's $\psi_{q(\text{min})} = \pi/4 + \phi/2$. Because of this fact the upper bound bearing capacity in the velocity field method where the ground self weight is disregarded agrees with the plasticity solution obtained with the ground self weight similarly disregarded.

(2) Stepping shape and bearing capacity characteristics

Fig. 12 shows the relation between the increase in stepping height ration and the variation in the bearing capacity, with the stepping width ration set to the normal value of $n = 0.3$, where the slope angle $\beta = 30^\circ, 45^\circ$. With $n = 0.3$, the failure mechanisms will be mechanism A and mechanism B, but the bearing capacity characteristics can be regarded as the same because mechanism A' changes with mechanism A, and the bearing capacity value changes continuously.

White failure mechanisms A and A' are demarcated with $(1 - n - m \cdot \cot \phi) = 0$ as the boundary, when the stepping height is noted in the case where the stepping width ratio "n" has been determined, the stepping height ratio $m_{cr}$ at the boundary is shown by the following equation:
\[ m_{cr} = (1-n)\tan \phi \approx (1-n) \cdot \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right) \]  \hspace{1cm} (22)

That is if \( m < m_{cr} \), mechanism A will occur, and if \( m \geq m_{cr} \), mechanism A' will occur, with the two bearing capacities varying continuously.

This reveals that mechanism A, which permits the foundation to move horizontally, has its bearing capacity decrease as the stepping height ratio "m" increases.

Next, Fig. 13 shows the relation between the stepping width ratio "n" and the bearing capacity. This explains that, in the case where the lower edge of the stepping can be accommodated in the slip plane, the variation in the stepping width affects the bearing capacity very little.

2.3 Failure mechanism of a stepped footing subjected to inclined load and the bearing capacity$^{5}$

Kezdi$^{11}$ has already shown that a active wedge angle \( \phi \) is determined uniformly at a certain load inclination angle \( \alpha \) if the ground self weight is not present (\( \gamma = 0 \)) and the foundation bottom is flat (\( m = 0, n = 0 \)). Based on the slip-line having this \( \phi \), a failure mechanism and admissible velocity field can be assumed and the bearing capacity obtained when an inclined load is at work. However, it is apparent that the upper bound bearing capacity value obtained from this method is less accurate than that obtained when the
ground self weight and the stepping height are taken into consideration. In the following paragraphs, we use failure mechanisms and admissible velocity fields under inclined loads with the ground self weight and stepping shape taken into account. In addition, we will propose more generalized and adaptable bearingcapacity equations. Comparisons are also made with the commonly used conventional methods that can be established under the special conditions for this calculation formula \((\gamma=0, m=0, n=0)\). The failure mechanism A will be used.

2. 3. 1  Failure mechanism in the presence of an inclined load

The steady-state conditions for the admissible velocity field are as follows in the presence of an inclined load:

1. Displacement vector \(V_A\) in the foundation due to the inclined load

A foundation on a slope accompanies a movement toward the valley side in all the cases. Under the action of loads whose direction is only perpendicular, this influence can be safely disregarded because no external work is done even if the foundation moves horizontally. This has already been considered. Because a component force also exists under an inclined load in the horizontal direction, consideration must be given to the admissible velocity field regarding the horizontal movement of the foundation.

In the present discussion, the displacement velocity in the displacement direction of the foundation is expressed as \(V_A\), as shown in Fig. 14.

When the displacement velocity component in the inclined external force direction is expressed as \(V_0\), the following equation can be established as a relation between \(V_A\) and \(V_0\):

\[
V_0=V_A \cdot \cos(\delta - \alpha)
\]

![Fig. 14 Failure mechanism under the inclined load, and admissible velocity field (mechanism A)](image-url)
Continuous condition in "bc" and "oa"

The condition must be continuous for the components of \( V_\theta \) and \( V_1 \) vertical to the "bc" and "oa" planes. Therefore,

\[ V_\theta \cdot \cos \delta = V_1 \cdot \sin (\phi - \phi) \]

Steady-state conditions in "ab" plane

The foundation in the "ab" plane and its adjacent ground will not be superposed with the foundation, but in such a condition that they are in contact at all times or separated. Therefore, the following equation should be established between the horizontal velocity components of the foundation and the ground:

\[ V_\theta \cdot \sin \delta \geq V_1 \cdot \cos (\phi - \phi) \]

And when the steady-state conditions for the perpendicular direction is considered at the same time, the following equation:

\[ V_\theta \cdot \sin \delta \geq V_\delta \cdot \cos \delta \cdot \cos (\phi - \phi) / \sin (\phi - \phi) \]

that is, a conditional equation between \( \delta \) and \( \phi \) that meets \( \tan \delta \geq 1 / \tan (\phi - \phi) \) is required.

If the foundation and the ground will move integrally, the following conditional equation can be considered between \( \delta \) and \( \phi, \phi \), as has been discussed in Section 2.1:

\[ \tan \delta \cdot \tan (\phi - \phi) = 1 \] (23)

Continuous condition on both sides of the transition region "oef"

This is expressed by equation (14) the same way as the condition with the perpendicular load acting upon.

Thus, the admissible velocity field has been determined as described above. The internal dissipation energy along the discontinuity lines of each velocity can then be easily obtained in the same manner as for a perpendicular load.

The total internal dissipation energy is shown in the following equation:

\[ E_{\text{total}} = c \cdot B \cdot N_c \cdot V_\delta \cdot \cos \delta \]

where \( N_c \) is the same as the \( N_c \) of the perpendicular load given in Eq. (16).

The external work can also be obtained in the same manner as for a perpendicular load.

The sum \( \Sigma W \) of the external work due to the dead weight of soil clumps is shown as follows:

\[ \Sigma W = - \gamma \cdot B^2 / 2 \cdot N_r \cdot V_\delta \cdot \cos \delta \]

where \( N_r \) is the same as the \( N_r \) for perpendicular loads given in Eq. (17). The external work due to the inclined load "q" is obtained relative to the external force direction \( V_\theta \).

And the following equation can be shown by substituting \( V_\delta \) into the formula and taking the displacement direction of the foundation into account:

\[ q \cdot B \cdot V_\delta \cdot \cos (\delta - \alpha) \]

Therefore, the total external work is shown by the following equation:

\[ W_{\text{total}} = q \cdot B \cdot V_\delta \cdot \cos (\delta - \alpha) - \gamma \cdot B^2 / 2 \cdot N_r \cdot V_\delta \cdot \cos \delta \]

When "q" is sought from the condition \( W_{\text{total}} = E_{\text{total}} \) the following equation is obtained:
\[ q = (c \cdot N_c + \frac{r \cdot B}{2} \cdot N_r) \cdot \frac{\cos \delta}{\cos(\delta - \alpha)} \]  \hspace{1cm} (24)

The "q" thus obtained is a bearing capacity in the direction of the load inclination angle \( \alpha \), and the bearing capacity in the perpendicular direction is

\[ q_v = q \cdot \cos \alpha \]

\[ = (c \cdot N_c + \frac{r \cdot B}{2} \cdot N_r) \cdot \frac{\cos \delta \cdot \cos \alpha}{\cos(\delta - \alpha)} \]  \hspace{1cm} (25)

thus, the evaluation formula is obtained for the perpendicular bearing capacity in the presence of an inclined load.

In this case, \( N_c, N_r \) are the bearing capacity coefficients at the inclined load obtained from Eqs. (16) and (17), and the minimum value for the bearing capacity can be obtained by varying the active wedge angle \( \phi \).

Since, for all of these, the displacement direction angle \( \delta \) of the foundation is established under a condition that the ground and the foundation move integrally, Eq. (23) must have been satisfied.

While this paragraph described the evaluation equations for the bearing capacity under the inclined load relative to mechanism A, the same principles apply to mechanism A'. This can be dealt with by means of substituting \( N_c \) and \( N_r \) in Eq. (25) into Eqs. (20) and (21) for mechanisms \( N_c \) and \( N_r \) in mechanism A'.

2. 3. 2 Result of bearing capacity calculations

Fig. 15 shows the relation between the inclined load angle and the wedge angle. The figure shows the relation that can be obtained as an analytic equation in the case where the

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Fig. 15  Relation between the inclined load angle and the active wedge angle

1. Simplified method \( \psi = 45^\circ + \phi 12 - \alpha \)
2. \( \psi (\beta = 45^\circ) \) based on an analytic equation \( (\gamma = 0) \)
3. \( \psi (\beta = 30^\circ) \) based on an analytic equation \( (\gamma = 0) \)
4. \( \psi_{ev(s, n)} (\beta = 45^\circ) \) based on the velocity field method
5. \( \psi_{ev(s, n)} (\beta = 30^\circ) \) based on the velocity field method

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\( B = 5 \text{m} \)
\( m = 0.3 \)
\( n = 0.3 \)
\( \beta = 30^\circ, 45^\circ \)
\( \beta_1 = 73^\circ \)
\( \gamma = 2 \text{tf/m}^3 \)
\( c = 20 \text{tf/m}^3 \)
\( \phi = 30^\circ \)
ground self weight and the stepping shape are disregarded, and the wedge angle $\varphi_{\text{qv(min)}}$ giving the minimum bearing capacity from the bearing capacity Eq. (24), which takes the ground self weight and the stepping shape into consideration. Further, the wedge angle ($\phi = \pi/4 + \phi/2 - \alpha$) is also shown as a practical simplification.

Similarly, with what has been discussed on the perpendicular load, the wedge angle $\phi$ based on the analytic equation of Kezdi and $\varphi_{\text{qv(min)}}$ based on the velocity field method yield different results due to the ground self weight of and the stepping shape. Fig. 16 shows the relation between $\phi$ as obtained from the analytic equation and $\varphi_{\text{qv(min)}}$ as based on the velocity field method in the case where the ground self weight and the stepping shape are disregarded. Both are in agreement irrespective of the inclined load when the ground self weight is not present ($\gamma = 0$) and the stepping height ratio is $m = 0$. The reason for this is the same as that discussed regarding the perpendicular load action.

Fig. 15 shows a case where the stepping height ratio $m = 0.3$, a value used frequently in actual applications. For the scope requiring consideration on the inclined load, hori-
3. Basic characteristics of the bearing capacity and verification of bearing capacity formulas

This section discusses failure mechanisms and bearing capacity characteristics based on loading test of bearing capacity using an indoor small size model and a in-situ large-size model. Previously proposed bearing capacity equations are also verified. The indoor loading test of bearing capacity used two methods: a gravitational field capable of detailed investigation into the basic failure properties, and a centrifugal model capable of an loading test dealing with various foundation widths in a small-size model to satisfy the approximation law of stress. The ground used in the gravitational field model consists of a soil using marine clay that simulates soft rock as the main material. The ground used in the centrifugal model consists of sandy soil, comprising the standard Toyoura sand, and undisturbed Shirasu, which was blocksampled from the site. Therefore, the centrifugal model makes it possible to run large-scale in-situ loading test of bearing capacity using the same Shirasu, and to compare the bearing capacities by approximating the pressure levels.

Since the details of these loading tests have already been reported**, only brief summaries are given in the sections that follow. The indications of stepping shapes (B, m, n) are identical with those in Fig. 2.

3. 1 Progress of failure and failure mechanisms

3. 1. 1 Loading test of bearing capacity in a gravitational field model using soil cement

Photos. 1 (a), and (b) are representative states of failure observed in a series of the loading tests under a perpendicular load in which (a) shows the case of (m=0.5, n=0.5) and (b) the case of (m=1.0, n=0.3). The test specimens had a footing width B=10 cm
and a depth $L=40$ cm. The loading test soil tank used acrylic plates with a thickness of 60 mm to assure sufficient rigidity. Conditions could thus be considered two-dimensional.

Photo. (a) for $(m=0.5, n=0.5)$ shows a failure mechanism A nearly identical to the one proposed earlier. Rigid wedges are developed on the lower face of the stepping, as are the rigid in the passive regions, but the failure plane extended deep into the slope direction, and thus the transient region decreased somewhat.

Photo. (b) for $(m=1.0, n=0.3)$ shows the failure mechanism A', in which a active region has been formed in the lower part of the stepping, and transition and passive region are formed. Because the footing moved horizontally, a clearance was developed between the footing on the upper part of the stepping and the soil masses, the region being divided into two rigid bodies. The slip line is shallow as a whole, with only a small scope of influence.

Fig. 17 shows the comparison of failure shapes relative to $\beta = 45^\circ$ and $60^\circ$ with the footing shape $m=0.5$ and $n=0.5$ to see the influence of the slope angle $\beta$. In both cases, it can be seen that cracks have developed below the footing off to the right. A basic failure mechanism has been formed in the lower part of the stepping due to the active wedge region, transition region, and passive region. In the lower stepping, wedges have grown deep at $\beta=60^\circ$, but slipped as if drawing a shallow arc at $\beta=45^\circ$.

Fig. 18 shows the failure shapes when the margin width was taken $\rho=0.5$ in the footing shape $m=0.5$, $n=0.5$, and $\beta=45^\circ$. Similarly with those in Fig. 17, it can be seen that cracks have developed below the footing to the right, and active wedges are formed in the lower part of the stepping, with wedges trying to pierce in fairly deep.

Fig. 19 shows the difference between the footing shapes, where footings with $m=1.0$ and $n=0.3$, and $m=0.2$ and $n=0.3$ are used with a slope angle of $\beta=45^\circ$. Also in these cases the cracks have appeared below both footings to the right. Actively slipped soil masses are generated in the upper part of the stepping, having a higher stepping height of $m=1.0$. Further, these soil masses and the foundation footing have behaved integrally. Active wedges have appeared also in the stepping bottom, resulting in a small slippage as a whole, while in the stepping having a lower height of $m=0.2$, a single active wedge developed in the lower face of the footing, resulting in large overall slippage as a whole. The differences in failure mechanisms due to stepping shape differences are apparent from these phenomena.

According to the above observation on the progress of the failure, we found that the following two patterns of failure exist.

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**Fig. 17** Difference in final failure shapes due to slope angle: soil cement

**Fig. 18** Final shape when a front margin width is provided: soil cement

**Fig. 19** Difference in final failure shapes due to stepping height: soil cement
In the first pattern, a single active wedge is generated at the lower part of the foundation, a fan-shaped transition region is formed adjacent to it, and a passive region is formed on the slope side, which takes place in the stepping with lower height. This failure mechanism correlates to failure mechanism A proposed earlier.

In the second pattern, the displayed soil appears halfway up the stepping, with a active wedge region, fan-shaped transition region, and passive region formed below the lower face of the stepping. This pattern takes place in the stepping with higher height. This failure mechanism explains the failure mechanism A’ proposed earlier. Slip-lines from the specific point surrounded by the footing were not observed. This proves the reasonability of the assumed failure mechanisms.

### 3.1.2 Bearing capacity with centrifugal model loading tests using sandy soil

Figs. 20 through 22 show the loading tests of bearing capacity with a scale model of 1/60 (B = 5 cm) using the standard Toyoura sand to estimate the slip-lines from the displacement vectors indicated by X-ray photography of metallic lead markers embedded in the ground.

Fig. 20 shows the influence of the stepping width ratio "n" at a slope angle β of 30° for wide stepping (n = 0.8) and narrow stepping (n = 0.4). The size of the plastic region is

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**(a) m = 0.4, n = 0.8**  
**(b) m = 0.4, n = 0.4**

**Fig. 20** Difference in displacement vectors with stepping width ratio "n": standard Toyoura sand

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**(a) m = 0.4, n = 0.6**  
**(b) m = 0.2, n = 0.6**

**Fig. 21** Difference in displacement vectors with stepping height ratio "m": standard Toyoura sand
almost constant regardless of the stepping width, as shown in the figure. Fig. 21 shows
the influence of the stepping height ratio "m" at a slope angle $\beta$ of 30°, (a) showing a high
stepping ($m=0.4$) and (b) low stepping ($m=0.2$). The size of the plastic region varies
with the stepping height. Where the stepping height is low, movements of the ground
agree almost well with failure mechanism A, as shown in the figure. Where the stepping
height is high, the failure characteristics are somewhere between the failure mechanisms A
and A', as indicated by the displacement vectors of the ground. The failure mechanisms
should both be attributable to the fact that the stepping shape is located at the boundary
region. If the stepping height is made still higher, the failure characteristic would
transform to the failure mechanism A'. Fig. 22 shows the case of the foundation having two-
step stepping, where no slip-line generation is seen from the specific point halfway up the
stepping. This fact shows that the stepping width and the number of steps do not influence
the bearing capacity, as discussed in 2.2.3.

3.1.3 Large-size model in-situ loading test with Shirasu

This section describes the failure mechanisms and bearing capacity characteristics
obtained from large-size model loading tests. The model, having a footing width $B=60$
cm and depth \( L = 600 \) cm, can be approximately two dimensional, was placed on nearly uniform natural ground with thick accumulation of Shirasu, a sandy volcanic ash, as shown in Fig. 23.

(1) Failure mechanism

Fig. 24 is a sketch of the slip line directly below a Type A footing with a flat foundation bottom. The concrete is placed directly on the ground to form the basis of the model for the bottom face of the footing. The wedge angle \( \phi \) formed directly below the foundation is approximately 68°. The internal friction angle corresponding to the Prandtl’s active wedge of \( \phi = 45° + \phi/2 \) is \( \phi = 46° \). This agrees very well with the internal friction angle \( \phi \) (PSC; \( \delta = 90° \)) = 45° to 47° at \( \delta = 90° \), as obtained by the plane strain compression test.

As has been already indicated by Kimura et al., the above results lend credence to large-size models in which when the bottom face is coarse the wedge develops a active shape of the Prandtl’s type. Section 2.2 shows that the active wedge angle that gives the minimum bearing capacity in the case when the stepping has no height \( (m = 0, n = 0) \) and that is obtained disregarding the ground self weight \( (\gamma = 0) \) agrees the conventional analytic equations and the velocity field method proposed in this thesis. The angle \( \delta = 45° + \phi/2 \). In the present loading test, the ground self weight term \( N_r \) in the bearing capacity coefficient at a slope angle of \( \beta = 45° \) became extremely small, and the influence of the ground self weight grew next to nil. Fig. 24 proves that the theoretical estimations are verified experimentally.

Figs. 25 (a), (b), and (c) show the shapes of slip lines in each test specimen and the locations of tips of the slip lines appeared on the slope surface, as observed after the loading test. These figures also show the slip lines calculated according to the velocity field method proposed in this thesis and the active wedge angle \( \phi \) generated from the footing edge (mountain side) in the upper part of the stepping.

In order to take note of the shape of the active wedge directly below the footing, when the angle made by the accumulation face on the ground and the main stress is \( \delta = 90° \) (Fig. 24), the internal friction angle is assigned a value of \( \phi = 46° \) when calculating the slip line.
shapes. As can be seen from the figures, with Type B-1 stepping having a low height (also Type A), the slip line goes through a location fairly deep in the slope, but with an increase in the stepping height, the triangle riadai wedge directly below the footing grows smaller, and the slip line passes through a shallow location. In this case, a rigid region integrated with footing is formed between the stepping and the footing bottom face. It has been theoretically clarified that the active wedge angle varies with the ground self weight as well as the stepping shape, and the higher the stepping height the smaller the active wedge angle $\phi$. These conclusions have also been shown experimentally.

Fig. 25 (d) shows the failure of the ground in Type C stepping, which has two steps. That there are no slip-lines generated from halfway stepping supports the concept adopted in this thesis that the number of steps does not influence the bearing capacity.

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**Fig. 25(a)** Slip line sketch Type-B-1 ($m=0.3, n=0.3$)

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**Fig. 25(b)** Slip line sketch Type-B-2 ($m=0.5, n=0.3$)
(2) Progress of failure and subgrade reaction

Figs. 26 (a), (b), and (c) show the relation between the perpendicular subgrade reaction and the footing settlement as measured by ground pressure gauges installed on the bottom of Type B-1, B-4, and C footings. The broken lines in the figures show an average subgrade reaction projected areas in the horizontal plane of the footings.

Generally, the subgrade reaction is concentrated on the horizontal part of the bottom of the footings. On the other hand, the perpendicular subgrade reaction built up on slanted sections of the footing appears to contribute very little to the bearing capacity. The subgrade force for the element 2 in Figs. 26 (a) and (b) and the elements 2 and 4 in Fig. (c), as the specific points of stress, are smaller than the element at the upper part of the stepping, making it difficult for a slip to occur from halfway the stepping. This fact indicates the reasonability of the failure mechanism discussed earlier.
3.2 Bearing capacity characteristics of shallow foundation and verification of bearing capacity formulas

3.2.1 Influence of stepping height

Fig. 27 shows the relation between the stepping height ratio "m" in Shirasu and the bearing capacity "q". The bearing capacity "q" decreases with the stepping height, illustrating the same trend as the result of the theoretical analysis given in Paragraph 2.2.3. The figure also shows mechanism A according to the velocity field method proposed in this thesis (formulas 15, 16, and 17) and the simplified method which further simplifies the velocity field method. The bearing capacity calculations agree well with the experimental values, signifying that the applicability of the bearing capacity formulas were verified. The shear strength constant used in the calculation is $c = 2.2\text{t/fm}^2$, $\phi = 43.2^\circ$ taking into account the anisotropy of the shear strength from the result of the plane strain compression test carried out in detail on undisturbed test specimens of in-situ Shirasu.

As has already been indicated, the shear constant should take the influence of progressive failure into account. However, in the present loading test (and also in actual slope foundation), because of the small main stress rotation, short slip line length, and the fact that Shirasu does not exhibit extreme strain softening, the influence of progressive failures has been disregarded.
In a supplementary position, IESM (Inter Element Slip Model)\textsuperscript{141} and RBSM (Rigid Body Spring Model)\textsuperscript{151} were used to analyze the bearing capacity. The analysis results are also shown in Fig. 27.

The figure also shows the results of a loading test\textsuperscript{160} in a centrifugal (20G) of a small-size model, 1/20 scale of the large-size model, using the same Shirasu. The tendency to decrease due to bearing capacity values and the stepping height ratio "m" were in near agreement in both models. This study is probably the first to compare loading tests on a 1) large-size in the gravitational field in-situ and 2) a small-size model in the centrifugal field. It is also the first to show the usefulness of such loading test in the centrifugal field.
3.2.2 Influence of stepping width and number of steps

Fig. 28 shows a relation between the stepping width ratio "n" and the bearing capacity "q". When bearing capacities of a stepped foundation and that of a flat foundation are compared, the bearing capacity in the stepped foundation when the slope angle \( \beta = 30^\circ \) is smaller by 30% to 50% than when the ground is flat (\( \beta = 0^\circ \)). The bearing capacity in the stepped foundation with \( \beta = 45^\circ \) is smaller by 40% to 70% than in flat ground (\( \beta = 0^\circ \)). The figure shows that changes in the stepping width ratio "n" at the respective slope angles \( \beta \) cause very little change in the bearing capacity, and thus the stepping width influences the bearing capacity only slightly.

While the figure also shows the results of a foundation having two steps, the bearing capacity differs little from that in other foundations where the sums of the stepping width and height becomes identical, its bearing capacity characteristics being unaffected by the number of steps. The causes for these results have already been discussed.

3.2.3 Influence of inclined load

Next, to elucidate the influence of inclined load, we ran a loading test\(^*\). To create inclined loads, we provided loading jacks with inclination angles. The loading test was performed on the case of the ratio of the perpendicular load to the horizontal load \( H \) being \( H/V = \tan \alpha = 0.1 \) and 0.2. The result is shown in Fig. 29.

It is seen from the figure that the ultimate bearing capacity decreases remarkably with an increase in the inclined load angle, the decreasing trend showing nearly the same gradient at the inclination angles \( \beta = 30^\circ \) and \( 45^\circ \), turning to a somewhat gentler gradient at \( \beta = 60^\circ \). This is because of the slope angle \( \beta \) largely influencing the bearing capacity with the increase in the inclined load angle \( \alpha \) as the slope angle \( \beta \) grows larger, this fact lending

Fig. 29  Relation between the inclined load angle and bearing capacity: Shirasu
credence to the theoretical discussions given in Paragraph 2.3.2. In the meantime, the bearing capacity in Fig. 29 represents the synthetic bearing capacity "q" in the inclined load direction in comparison with the experimental values, whereas the perpendicular component "q_v" of the bearing capacity generally required in designing is obtained from q_v = q * cos α.

The bearing capacity calculated value as obtained from the proposed formula (24) agrees well with the observed value also on inclined load. The shear strength constant used in this is the same as is used in Paragraph 3.2.1.

4. Conclusions

This thesis proposes bearing capacity formulas for shallow foundations on slopes, using the velocity field method that gives the upper bound bearing capacity value, and verified its applicability by means of indoor and in-situ loading tests.

These bearing capacity formulas are characterized in that they are based on an admissible velocity field that can simultaneously consider the influences of ① stepping shapes, ② ground self weight, and ③ inclined load. In addition, the formulas are highly adaptable to different conditions.

The basic matters that have been elucidated through the theoretical and experimental studies described in this thesis are:

(1) The bearing capacity of a shallow foundation with stepped footing on slopes can be expressed by formulas that represent failures in the entire foundation. To do this, the foundation is divided into a flat portion and a stepped portion and combining the basic failure mechanisms for each part. The formulas are described.

In the proposed bearing capacity formulas, the bearing capacity that takes into account influences from the stepping shapes, ground self weight, and load inclination can be derived by means of varying the active wedge angles.

(2) The model loading tests in laboratory in the gravitational field and centrifugal field and in-situ large-size model loading tests confirmed that the bearing capacity decreases with increases in the stepping height, slope angle, and inclined load angle, and that the stepping width and the number of steps have very little effect on the bearing capacity. These experimental results agreed well with our theory.

In addition, the bearing capacity characteristics as revealed by the centrifugal loading test agreed well with the in-situ large-size model loading test having a similarity in the stresses, and their usefulness was verified.

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